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PHYSICS

ON THE MALDACENA TYPE CONJECTURE IN RELATION WITH SCALE RELATIVITY THEORY

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A Maldacena type conjecture in relation with Scale Relativity Theory using Schrödinger representations of fractal type dynamics of any physical system is proposed. The operational procedure developed by the authors is based on the phase coherences of the dynamics, involving, through dual "holographic implementations", one in the usual space and the other in scales space, apolar transport of the temporal cubics of fractal type. In the Riemannian spaces associated with the cubics, their apolar transport becomes equivalent to the parallel transport of the hyperbolic planes. In this way, it is shown that any dynamics which reflects SL (2R) "processes" has intrinsic hyperbolic geometries, harmonic mappings, from usual and scales spaces to the hyperbolic one, implying correlations between the General Relativity, in Ernst sense, and Skyrme Theory.

INTRODUCTION – A BREIF REMINDER ON JUAN MALDACENA'S CONJECTURE

Towards the end of the 90's, Juan Maldacena proposed a novel paradigm of the Universe. The model states that gravity can be generated by vibrating strings, so that a complete rethinking of the understanding of physics at both large and quantum scales was attempted. Briefly, the complex mathematical representation of these objects (strings) occurs in a 10-dimensional space. In this conjecture, the evolution of systems can be simplified by means of a holographic approach. The Maldacena conjecture [1] was later related to the string theory or M theory on particular mediums, such as $AdS_d \times M_{D-d}$ (where AdS_d is an anti de Sitter space of space-time dimension d, and M_{D-d} is a certain spatial reduction of dimension $D - d$ with $D = 10$ or 11 for the string theory or M theory, respectively). In other words, Maldacena's conjecture states that a quantum string- or M-theory in a medium like the earlier described one has as mathematical equivalent an invariant quantum field theory in a space-time (d – 1 – dimension), which is a boundary interpretation for AdSd. This novel interpretation also bridges the relation between the quantum and the classical theory. Several authors build on Maldacena's conjecture, offering a wider reach of the paradigm, as seen especially in the works of Gubser, Klebanov and Polyakov [2], and of Witten [3]. Maldacena conjecture makes use of the holography property as a general characteristic of all physical systems. Assuming that this conjecture is valid, it can offer information of non-perturbative nature on ordinary invariant quantum field theories. The reach of Maldacena' conjecture ranges from N field theories to the string/ M-theory or even supergravity. This property of the model will also permit a larger impact on the non-perturbative M-theory. A similar paradigm can be implemented for more complex systems, like the fractal ones. Transition from the boundary to the core of the systems by maintaining intact the dynamical information of the system represents a staple of fractal physics. Also, in the Scale Relativity Theory according to Nottale, quantum mechanics becomes a particular case of fractal mechanics of fractal dimension $D_F = 2$.

In the present work, the principle proposed by Maldacena will be extended, and the possibility of a holographic conjecture in relation with Scale Relativity Theory in the form of Fractal Theory of Motion which becomes operational will be analyzed.

MATHEMATICAL MODEL

"HOLOGRAPHIC IMPLEMENTATIONS" OF PHYSICAL SYSTEM DYNAMICS ON BOTH THE USUAL AND SCALES SPACE

The common models used to describe physical system dynamics in the usual space have been based either on a combination of basic theories, derived especially from physics [4–6], or on computer simulations [7–9]. In such a context, their description has involved both computational simulations based on specific algorithms [7–9] and developments on standard theories. With respect to the models developed on standard theories, the following classes can be distinguished: a class of models developed on spaces with integer dimension – *i.e.*, differentiable models (for example Classical Mechanics, Quantum Mechanics, etc.) [10–12], and a class of models developed on spaces with non-integer dimensions, which are explicitly written through fractional derivatives [8,9] – *i.e.*, non-differentiable models, with examples including the fractal models.

Expanding the previous class of models, new developments have been made, based on Scale Relativity Theory, either in monofractal dynamics, as in the case of Nottale [10], in fractal dimension D_F=2, or in fractal dynamics, as in the case of the Fractal Theory of Motion [12,13].

Both in the context of Scale Relativity Theory according to Nottale [10] and in that of Fractal Theory of Motion [12,13], the fundamental hypothesis has been the following: supposing that any physical system was assimilated both structurally and functionally to a fractal object, the said dynamics can be described through motions of any physical system entity, dependent on the chosen scale resolution, on continuous and non-differentiable curves (fractal curves).

Such a hypothesis may be illustrated by considering the following scenario in the dynamics of a physical system: between two successive interactions of the entity belonging to any physical system, the trajectory of the physical system entity is a straight line. This line becomes non-differentiable in the impact point. Considering that all interaction points formed an uncountable set of points, there results that the trajectories of any physical system entity become continuous and non-differentiable (*i.e.*, fractal curve). Actually, reality has been much more complicated, taking into account both the diversity of the entity which composes any physical system and also the various interactions between them. Extrapolating the previous reasoning to any physical system, there results that it can be assimilated to a fractal.

All these considerations show that, in the description of any physical system dynamics, instead of “working” with a single variable (regardless of its nature, *i.e.*, velocity, density, etc.) described by a strict non-differentiable function, it is possible to “work” only with approximations of this mathematical function, obtained by averaging them on different scale resolutions. As a consequence, any variable aimed at describing any physical system dynamics will perform as the limit of a family of mathematical functions, being non-differentiable for null scale resolutions and differentiable otherwise [10,11]. To put it differently, from a mathematical point of view, these variables could be explained through fractal functions, *i.e.* as functions dependent not only on spatial and temporal coordinates, but also on scale resolution.

For a large temporal scale resolution with respect to the inverse of the highest Lyapunov exponent [14,15], the deterministic trajectories of any entity belonging to a physical system dynamics can be replaced by a collection of potential (“virtual”) trajectories. Thus, the concept of definite trajectory can be substituted by that of probability density. Based on all the above considerations, the fractality expressed through stochasticity, in the description of the dynamics of any physical system dynamics, becomes operational in the fractal paradigm through the Fractal Theory of Motion. Various applications of this model have been described in literature [16–20].

All these observations evidence that any description of physical system dynamics requires simultaneously dynamic descriptions, in relation with the Fractal Theory of Motion, on two manifolds: one on the usual space, and the other one on the scales space. Accordingly, the fundamental assumption of the here proposed model is that the dynamics of any entity of the physical system will be described by continuous but non-differentiable motion curves (fractal motion curves). These fractal motion curves exhibit the property of self-similarity in every point, which can be translated into a property of holography (every part reflects the whole and *vice versa*) [10,11]. Basically, the discussion will be about “holographic implementations” of the dynamics of any physical systems, in relation with the Fractal Theory of Motion [12,13].

“HOLOGRAPHIC IMPLEMENTATIONS” OF PHYSICAL SYSTEM DYNAMICS ON THE USUAL SPACE

Let us consider that the scale covariance principle (the physics laws applied to the dynamic descriptions of any physical system are invariant with respect to scale resolution transformations [10]) and postulate that transition from the standard (differentiable) dynamics to the fractal (non-differentiable) dynamics of any physical system can be implemented by replacing the standard time derivative $\frac{d}{dt}$ by the non-differentiable operator $\frac{\hat{d}}{dt}$ [12,13]:

$$\frac{\hat{d}}{dt} = \partial_t + \hat{V}^l \partial_l + \frac{1}{4}(dt)^{\left(\frac{2}{D_F}\right)-1} D^{lp} \partial_l \partial_p \tag{1}$$

where:

$$\begin{gathered} \hat{V}^l = V_D^l - V_F^l \\ D^{lp} = d^{lp} - i\bar{d}^{lp} \\ d^{lp} = \lambda_+^l \lambda_+^p - \lambda_-^l \lambda_-^p \\ \bar{d}^{lp} = \lambda_+^l \lambda_+^p + \lambda_-^l \lambda_-^p \\ \partial_t = \frac{\partial}{\partial t}, \partial_l = \frac{\partial}{\partial X^l}, \ \partial_l \partial_p = \frac{\partial}{\partial X^l}\frac{\partial}{\partial X^p}, i = \sqrt{-1}, \quad l,p = 1,2,3 \end{gathered} \tag{2}$$

In the above relations, $\hat{V}^l$ is the complex velocity, V_D^l is the differentiable velocity independent on scale resolution dt and V_F^l is the non-differentiable velocity dependent on scale resolution. X^l is the fractal spatial coordinate and *t* is the non-fractal time, acting as an affine parameter of the motion curves. D^{lp} is the constant tensor associated with the differentiable-non-differentiable transition of dynamic processes, λ_+^l is the constant vector associated with the forward differentiable-non-differentiable of the dynamic processes, and λ_-^l is the constant vector associated with the backwards differentiable-non-differentiable of dynamic processes. D_F is the fractal dimension of the movement curve. For the fractal dimension, it is possible to choose any definition: Kolmogorov fractal dimension, Hausdorff-Besikovici fractal dimension, etc. [11], yet, once chosen, this becomes operational, having to be constant and arbitrary: $D_F < 2$ for correlative dynamic processes, $D_F > 2$ for non-correlative dynamic processes, etc. [10,11].

Now, the non-differentiable operator plays the role of the scale covariant derivative, namely it is used to write the fundamental equations of the dynamics of any physical system, in the same form as in the classic (differentiable) case. Under these conditions, accepting the functionality of the scale covariant principle, *i.e.*, applying the scale covariant derivative (1) to the complex velocity field (2), in the absence of any external constraint, the geodesic equation of any physical system takes the following form [12,13]:

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i + \frac{1}{4}(dt)^{\left(\frac{2}{D_F}\right)-1} D^{lk} \partial_l \partial_k \hat{V}^i = 0 \tag{3}$$

This means that the fractal local acceleration $\partial_t \hat{V}^i$, the fractal convection $\hat{V}^l \partial_l \hat{V}^i$ and the fractal dissipation $D^{lk} \partial_l \partial_k \hat{V}^i$ of any physical system entity make their balance in any point of the motion fractal curve. Moreover, the presence of the complex coefficient of viscosity-type $4^{-1}(dt)^{\left(\frac{2}{D_F}\right)-1} D^{lk}$ in the physical system dynamics specifies that it is a rheological medium. So, any physical system has memory, as a datum, by its own structure.

If fractalization in the dynamics of any physical system is achieved by Markov-type stochastic processes, which involve Lévy type movements [10,11,14,15] of the physical system entities, then:

$$\lambda_+^i \lambda_+^l = \lambda_-^i \lambda_-^l = 2\lambda \delta^{il} \tag{4}$$

where λ is a coefficient associated to the differentiable-non-differentiable transition, and δ^{il} is Kronecker's pseudo-tensor.

Under these conditions, the geodesic equation takes the simple form:

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_l \hat{V}^i - i\lambda (dt)^{\left(\frac{2}{D_F}\right)-1} \partial^l \partial_l \hat{V}^i = 0 \tag{5}$$

For irrotational motions of the physical system entities, the complex velocity field $\hat{V}^i$ takes the form:

$$\hat{V}^i = -2i\mathcal{D}_u \partial^i \ln \Psi, \quad \mathcal{D}_u = i\lambda (dt)^{\left(\frac{2}{D_F}\right)-1} \tag{6}$$

Then, by substituting (6) in (5), the geodesic equation (5) (for details see the method discussed in [15–19]) becomes a Schrödinger-type equation at various scale resolutions (Schrödinger equation of fractal type):

$$\mathcal{D}_u^2 \partial^l \partial_l \Psi + i\mathcal{D}_u \partial_t \Psi = 0 \tag{7}$$

Variable $\Phi = -2i\mathcal{D}_u \ln \Psi$ defines, through (6), the complex scalar potential of the complex velocity field, while Ψ corresponds to the state function of fractal type. Both variables, Φ and Ψ, have no direct physical meaning, but possible "combinations" of them can be acquired, if they satisfy certain conservation laws.

Let us make explicit such a situation for Ψ. To this end, it is first noticed that the complex conjugate of Ψ, that is $\overline{\Psi}$, satisfies, through (7), the following equation:

$$\mathcal{D}_u^2 \partial^l \partial_l \overline{\Psi} + i\mathcal{D}_u \partial_t \overline{\Psi} = 0 \tag{8}$$

Multiplying (7) by $\overline{\Psi}$ and (8) by Ψ, subtracting the results and introducing notations:

$$\rho = \Psi\overline{\Psi}, \quad \boldsymbol{J} = i\mathcal{D}_u(\Psi\nabla\overline{\Psi} - \overline{\Psi}\nabla\Psi) \tag{9}$$

it is possible to obtain the conservation law of states density of fractal type:

$$\partial_t \rho + \nabla J = 0 \tag{10}$$

In (10), ρ corresponds to states density of fractal type and $\boldsymbol{J}$ corresponds to the state density current of fractal type.

"HOLOGRAPHIC IMPLEMENTATIONS" OF PHYSICAL SYSTEM DYNAMICS ON THE SCALES SPACE

Let us consider a fractal function $F(x)$ in the interval $x \in [a, b]$, that describes any physical system dynamics. Then, the sequences of values for x:

$$x_a = x_0, x_1 = x_0 + \varepsilon, \dots, x_k = x_0 + k\varepsilon, \dots, x_n = x_0 + n\varepsilon = x_b \tag{11}$$

will correspond to $F(x, \varepsilon)$ as the broken line that connects the points:

$$F(x_0), \dots, F(x_k), \dots, F(x_n) \tag{12}$$

This broken line is an ε-scale approximation of $F(x)$, *i.e.* $F(x, \xi)$.

Let us consider another scale, and its $\bar{\varepsilon}$-scale approximation of $F(x, \bar{\varepsilon})$. Because $F(x)$ is a fractal function, it is self-similar almost everywhere, which can be translated into a property of holography (every part reflects the whole and *vice versa*) [10,11]. Basically, the topic of "holographic implementations" of the physical system dynamics in the scale space becomes operational, too. In such a context, the same result can be obtained if ε and $\bar{\varepsilon}$ are sufficiently small. Comparing these two approximations, an infinitesimal increase/ decrease $d\varepsilon$of ε corresponds to an infinitesimal increase/ decrease $d\bar{\varepsilon}$ for $\bar{\varepsilon}$. Therefore:

$$\frac{d\varepsilon}{\varepsilon} = \frac{d\bar{\varepsilon}}{\bar{\varepsilon}} = d\mu \tag{13}$$

Within this framework, the scale transition for $\varepsilon + d\varepsilon$ and $d\varepsilon$ must be invariant. There results:

$$\varepsilon' = \varepsilon + d\varepsilon = \varepsilon + \varepsilon d\mu \tag{14}$$

Performing (14) for $F(x,\varepsilon)$, the following equation can be obtained:

$$F(x,\varepsilon') = F(x,\varepsilon + \varepsilon d\mu) \tag{15}$$

Then,

$$F(x,\varepsilon') = F(x,\varepsilon) + \frac{\partial F}{\partial \varepsilon}(\varepsilon' - \varepsilon) \tag{16}$$

which then yields:

$$F(x,\varepsilon') = F(x,\varepsilon) + \frac{\partial F}{\partial \varepsilon}\varepsilon d\mu \tag{17}$$

Let us observe that, for an arbitrary fixed ε_0:

$$\frac{\partial \ln\left(\frac{\varepsilon}{\varepsilon_0}\right)}{\partial \varepsilon} = \frac{\partial(\ln\varepsilon - \ln\varepsilon_0)}{\partial\varepsilon} = \frac{1}{\varepsilon} \tag{18}$$

Thus, (16) can also be written as:

$$F(x,\varepsilon') = F(x,\varepsilon) + \frac{\partial F(x,\varepsilon)}{\partial \ln\left(\frac{\varepsilon}{\varepsilon_0}\right)} d\mu \tag{19}$$

Finally,

$$F(x,\varepsilon') = \left(1 + \frac{\partial}{\partial \ln\left(\frac{\varepsilon}{\varepsilon_0}\right)} d\mu\right) F(x,\varepsilon) \tag{20}$$

so that operator:

$$\widehat{D} = \frac{\partial}{\partial \ln\left(\frac{\varepsilon}{\varepsilon_0}\right)} \tag{21}$$

acts as a dilation/ contraction operator, depending on the given process. Thus, the invariance of equations that describe any physical system dynamics is expressed through, whether or not one of these equations is changed if the operator is applied, while specifying that the intrinsic variation of resolution is $\ln(\varepsilon/\varepsilon_0)$.

Thus, the spontaneous breaking of scale invariance of any fractal variable Q which shall describe any physical system dynamics implies the functionality of the following equation:

$$\frac{\partial Q}{\partial \ln\left(\frac{\varepsilon}{\varepsilon_0}\right)} = P(Q) \tag{22}$$

where $P(Q)$ is an arbitrary polynomial, associated to variable Q.

As a conclusion, in the scales space, any physical system dynamics can be described by means of two fundamental variables, namely, the first, the logarithms of resolutions and the second one, the scale time.

Considering all the above observations, since the scale space is now generalized to a non-differentiable and fractal geometry, various elements of the new description can also be used [10,12,13]:

i) Infinity of trajectories, leading to the introduction of a scale velocity field $\mathbb{V} = \mathbb{V}(\ln \mathcal{L}(\tau), \tau)$, where $\mathcal{L}$ is the non-differential space scale coordinate, and τ is the time scale coordinate;

ii) Decomposition of the derivative of the fractal space scale coordinate in terms of a "classical part" and a "fractal part", described by a stochastic variable, as in the case of the usual space, so that:

$$\langle d\xi_s^2 \rangle = 2\mathcal{D}_s d\tau \tag{23}$$

In (23), $\mathcal{D}_s$ is a constant coefficient assimilated to fractal-nonfractal transition in the scales space, and ξ_s is the fractal part of the differential spatial coordinate in the scales space;

iii) Introduction of the two-valuedness of this derivative because of symmetry breaking of the reflection invariance under the exchange $d\tau \leftrightarrow -d\tau$, leading to constructing a complex scale velocity $\widetilde{\mathbb{V}}$ based on this two-valuedness;

iv) Construction of a new total covariant derivative with respect to τ, which can be written as:

$$\frac{\hat{d}}{d\tau} = \frac{\partial}{\partial \tau} + \widetilde{\mathbb{V}} \frac{\partial}{\partial \ln \mathcal{L}} - i\mathcal{D}_s \frac{\partial^2}{(\partial \ln \mathcal{L})^2} \tag{24}$$

v) Introduction of a wave function as a re-expression of the action, which is now complex:

$$\Psi_s(\ln \mathcal{L}) = exp(iS_s/2\mathcal{D}_s) \tag{25}$$

In (25), Ψ_s represents the state function in the scale space, and S_s is the action in the scales space;

vi) Transformation and integration of the free Newtonian scale-dynamics equation:

$$\frac{d^2 \ln \mathcal{L}}{d\tau^2} = 0 \tag{26a}$$

in the form of a Schrödinger type equation now acting on scale variables:

$$\mathcal{D}_s^2 \frac{\partial^2 \Psi_s}{(\partial \ln \mathcal{L})^2} + i\mathcal{D}_s \frac{\partial \Psi_s}{\partial \tau} = 0 \tag{26b}$$

SOLUTIONS OF ONE-DIMENSIONAL SCHRÖDINGER EQUATIONS OF FRACTAL TYPE IN BOTH THE USUAL SPACE AND SCALES SPACE

The solution of the one-dimensional Schrödinger equation of fractal type in the compact form, both in the usual space and in the scales space, *i.e*:

$$\mathcal{D}_{u,s}^2 \partial_{l_{u,s}} \partial^{l_{u,s}} \Psi(x_{u,s}, t_{u,s}) + i\mathcal{D}_{u,s} \partial_{t_{u,s}} \Psi(x_{u,s}, t_{u,s}) = 0, \quad (27)$$

can be written in the form:

$$\Psi(x_{u,s}, t_{u,s}) = \frac{1}{\sqrt{t_{u,s}}} \exp\left(i \frac{{x_{u,s}}^2}{4\mu_{u,s} t_{u,s}}\right), \ \mu_{u,s} = \mathcal{D}_{u,s} \quad (28)$$

being defined, obviously, up to an arbitrary multiplicative constant. In the above relations, and also in the following ones, indexation with “u” defines the variables and parameters of dynamics in the usual space, while indexation with “s” defines the same variables and parameters of the same dynamics in the scales space.

As such, the general solution of equation (27) can be written as a linear superposition of the form:

$$\Psi(x_{u,s}, t_{u,s}) = \frac{1}{\sqrt{t_{u,s}}} \int_{-\infty}^{+\infty} u(y_{u,s}) exp\left[i \frac{(x_{u,s} - y_{u,s})^2}{4\mu_{u,s} t_{u,s}}\right] dy_{u,s} \quad (29)$$

Now, if $u(y_{u,s})$ is an Airy function of fractal type, then $\Psi(x_{u,s}, t_{u,s})$ will preserve this property, namely its amplitude is an Airy function of fractal type. Indeed, in this case:

$$u(y_{u,s}) \equiv Ai(y_{u,s}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} exp\left[i\left(\frac{\omega_{u,s}^3}{3} + \omega_{u,s} y_{u,s}\right)\right] d\omega_{u,s} \quad (30)$$

so that the state function (29) will be written in the form:

$$\Psi(x_{u,s}, t_{u,s}) = \frac{1}{2\pi\sqrt{t_{u,s}}} \int_{-\infty}^{+\infty} exp\left\{i\left[\frac{\omega_{u,s}^3}{3} + \omega_{u,s} y_{u,s} + \frac{(x_{u,s} - y_{u,s})^2}{4\mu_{u,s} t_{u,s}}\right]\right\} dy_{u,s} d\omega_{u,s} \quad (31)$$

If, at first, integration will be carried out after $y_{u,s}$, up to a multiplicative constant, the result is:

$$\Psi(x_{u,s}, t_{u,s}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} exp\left[i\left(\frac{\omega_{u,s}^3}{3} + \omega x_{u,s} - \mu_{u,s} t_{u,s} \omega_{u,s}^2\right)\right] d\omega_{u,s} \quad (32)$$

The final result, obtained based on a special relation developed in [29,30], is:

$$\Psi(x_{u,s}, t_{u,s}) = [Ai(k_{u,s} x_{u,s} - {v_{u,s}}^2 {t_{u,s}}^2)] exp\left[i v_{u,s} t_{u,s}\left(k_{u,s} x_{u,s} - \frac{2}{3} {v_{u,s}}^2 {t_{u,s}}^2\right)\right] \quad (33)$$

with:

$$\nu_{u,s} = k_{u,s}{}^{2}\mu_{u,s} \tag{34}$$

In these conditions, if Ψ is chosen in the form:

$$\Psi(x_{u,s}, t_{u,s}) = A(x_{u,s}, t_{u,s})\exp[i\phi(x_{u,s}, t_{u,s})] \tag{35}$$

where $A(x_{u,s}, t_{u,s})$ is an amplitude and $\Phi(x_{u,s}, t_{u,s})$ is a phase, by identifying in (33) the amplitude and the phase, there will result:

$$A(x_{u,s}, t_{u,s}) = Ai(k_{u,s}x_{u,s} - \nu_{u,s}{}^{2}t_{u,s}{}^{2}), \quad \phi(x_{u,s}, t_{u,s}) = \nu_{u,s}t_{u,s}\left(k_{u,s}x_{u,s} - \frac{2}{3}\nu_{u,s}{}^{2}t_{u,s}{}^{2}\right) \tag{36a,b}$$

Taking into account the asymptotic behavior of function $Ai(z)$ in its general form:

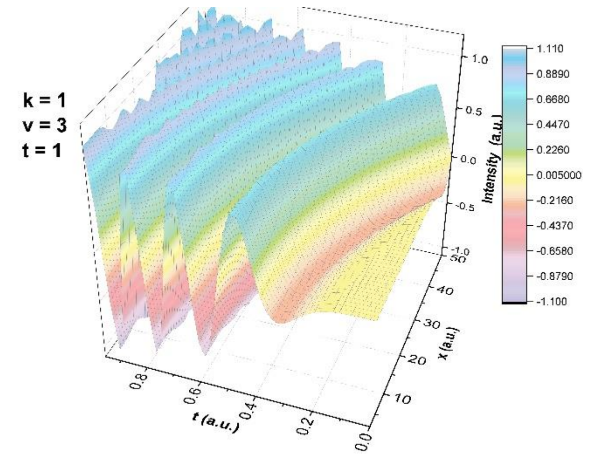
$$Ai(z_{u,s}) \sim \begin{cases} \frac{1}{2\pi^{1/2}} z_{u,s}{}^{-1/4} exp\left(-\frac{2}{3} z_{u,s}{}^{3/2}\right), & z_{u,s} \to +\infty \\ \frac{1}{\pi^{1/2}} |z_{u,s}|^{-1/4} \sin\left(\frac{2}{3}|z_{u,s}|^{3/2} + \frac{\pi}{4}\right), & z_{u,s} \to -\infty \end{cases} \tag{37}$$

the state (35) with (36a,b) function in the asymptotic limit $\Psi \to \Psi_A$ becomes:

$$\Psi_A \sim \begin{cases} \frac{1}{2\pi^{1/2}}(k_{u,s}x_{u,s} - \nu_{u,s}{}^{2}t_{u,s}{}^{2})^{-1/4} exp\left[-\frac{2}{3}(k_{u,s}x_{u,s} - \nu_{u,s}{}^{2}t_{u,s}{}^{2})^{3/2} + i\nu_{u,s}t_{u,s}\left(k_{u,s}x_{u,s} - \frac{2}{3}\nu_{u,s}{}^{2}t_{u,s}{}^{2}\right)\right] \\ \frac{1}{\pi^{1/2}}|k_{u,s}x_{u,s} - \nu_{u,s}{}^{2}t_{u,s}{}^{2}|^{-1/4} \sin\left[\frac{2}{3}|k_{u,s}x_{u,s} - \nu_{u,s}{}^{2}t_{u,s}{}^{2}|^{3/2} + \frac{\pi}{4}\right] exp\left[i\nu_{u,s}t_{u,s}\left(k_{u,s}x_{u,s} - \frac{2}{3}\nu_{u,s}{}^{2}t_{u,s}{}^{2}\right)\right] \end{cases} \tag{38}$$

Figure 1 plots the 3D and contour plot representation of the wave function, as defined through (35). We observe that the wave function can be influenced by a large series of factors, including time. For relative low values of the control parameters (t, v, k) the function follows closely the Airi type representation, which is dominant in (35). When the system evolved at a longer moment of time, a self-modulation of the system and a change in frequency are observed, as both time and the spatial coordinate are varied. Modulation appears on the temporal axis, defining a complex behavior on the spatial coordinate.

This means that, by simply selecting the appropriate scale (defined by: z, t, v, k), one can better investigate the particular dynamics of the system. Self-modulation is better seen when the properties of the wave are modified (through k), when we can observe that, for k = 6,v = 3 and t =1, the wavefunction defined the wave like structure in the space coordinate, while, in time, it complicates the features with multiple oscillation frequencies. This means that, by tailoring the resolution scale at which a system is investigated, we can obtain transition from a time- or space-modulated structure which characterizes particular phenomena. Further understanding of the space-time modulation of the wave function could become important when investigating transient phenomena like laser produced plasma, or complex fluid flows, where temporal analysis for a fixed spatial volume or spatial analysis for a fixed moment of time are often reported.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v = 3
t = 1
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x (a.u.)
t (a.u.)
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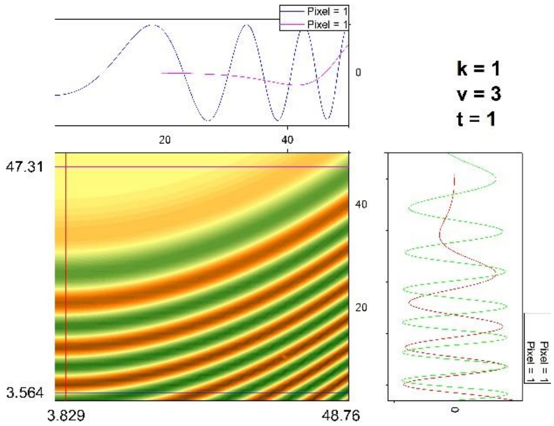
0.4470
0.2260
0.005000
-0.2160
-0.4370
-0.6580
-0.8790
-1.100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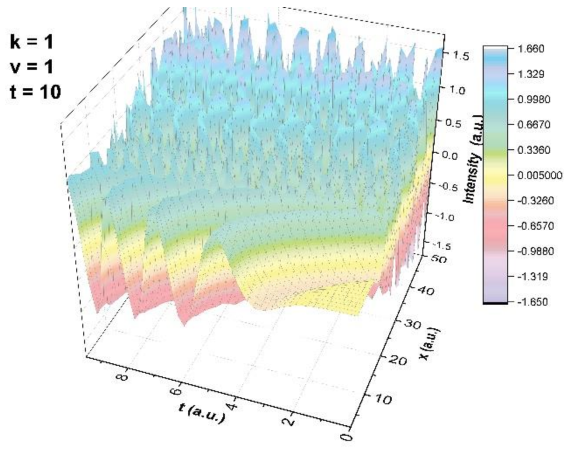
3.829
48.76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Intensity (a.u.)
x (a.u.)
t (a.u.)
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0.3360
0.005000
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-0.6570
-0.9880
-1.319
-1.650

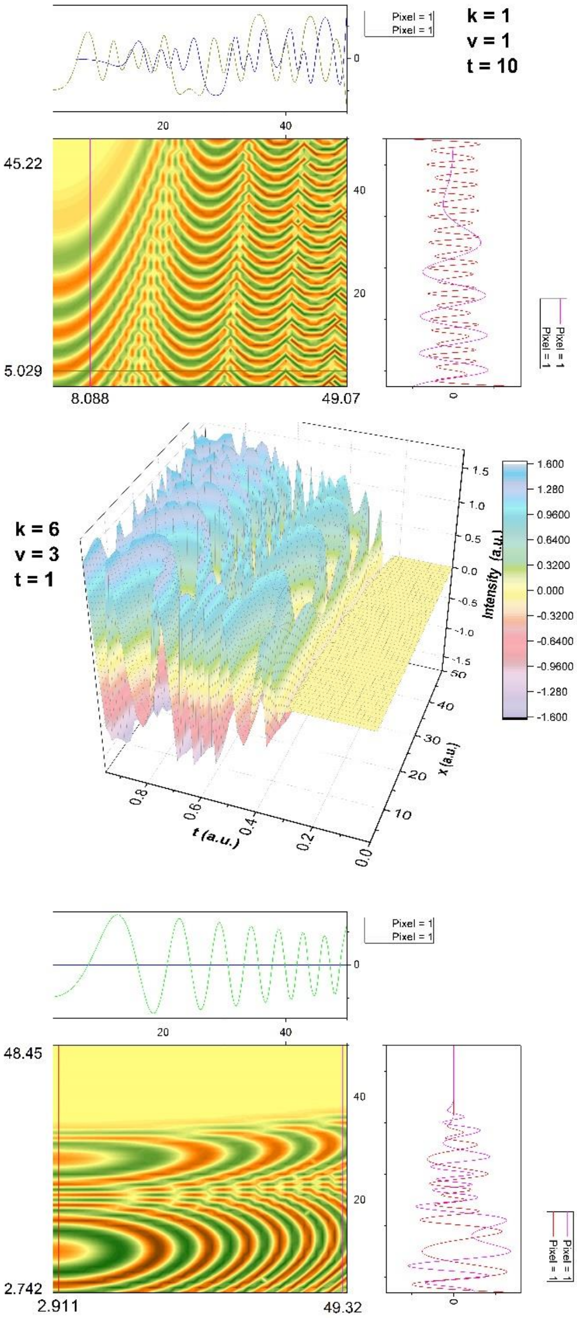

Figure 1.3D and contour plot representation of the wave function given through (35)

By substituting (35) in (27), through direct calculation, the following relation is checked:

$$i\partial_{t_{u,s}}\Psi + \mu_{u,s}\partial_{l_{u,s}}\partial^{l_{u,s}}\Psi = -\left[\partial_{t_{u,s}}\phi + \mu_{u,s}(\partial_{l_{u,s}}\phi)^2 - \mu_{u,s}\frac{\partial_{l_{u,s}}\partial^{l_{u,s}}A}{A}\right] + \frac{i}{2A^2}\left[\partial_{t_{u,s}}A^2 + 2\mu_{u,s}\partial_{l_{u,s}}(A^2\partial^{l_{u,s}}\phi)\right] \tag{39}$$

Now, the "specific constraints" necessary for Ψ to become a solution of the non-stationary differential equation (39) will be reducible to the differential equations:

$$\begin{gathered}\partial_{t_{u,s}}\phi + \mu_{u,s}\left(\partial_{l_{u,s}}\phi\partial^{l_{u,s}}\phi\right) = \mu_{u,s}\frac{\partial_{l_{u,s}}\partial^{l_{u,s}}A}{A}\\ \partial_{t_{u,s}}A^2 + 2\mu_{u,s}\left(A^2\partial_{l_{u,s}}\phi\right) = 0\end{gathered} \tag{40}$$

The first of these equations is the Hamilton-Jacobi equation of fractal type, while the second is the continuity equation of fractal type. From here, the correspondence with the hydrodynamic model of fractal type pertaining to Scale Relativity becomes evident, based on the following substitutions:

$$V_D^{i_{u,s}} = \mu_{u,s}\partial^{i_{u,s}}\phi, \quad \rho = A^2 \tag{41}$$

where $V_D^{i_{u,s}}$ is the differential component of the velocity field and ρ is the density of states. The conservation law of fractal type of the specific momentum can be found:

$$\partial_{t_{u,s}}V_D^{i_{u,s}} + V_D^{l_{u,s}}\partial_{l_{u,s}}V_D^{i_{u,s}} = -\partial^{i_{u,s}}Q \tag{42}$$

and, respectively, the conservation law of the density of states of fractal type:

$$\partial_{t_{u,s}}\rho + \partial^{l_{u,s}}\left(\rho V_D^{l_{u,s}}\right) = 0 \tag{43}$$

The specific potential of fractal type:

$$Q = -{\mu_{u,s}}^2\frac{\partial_{l_{u,s}}\partial^{l_{u,s}}\sqrt{\rho}}{\sqrt{\rho}} \tag{44}$$

through the induced specific force of fractal type:

$$f^{i_{u,s}} = -\partial^{i_{u,s}}Q = -{\mu_{u,s}}^2\partial^{i_{u,s}}\left(\frac{\partial_{l_{u,s}}\partial^{l_{u,s}}\sqrt{\rho}}{\sqrt{\rho}}\right) \tag{45}$$

becomes a measure of the fractal degree pertaining to the motion curves. In such a motion, the "specific constraints" (40) are also checked in detail, with a specific potential of a fractal type:

$$Q\left(x_{u,s}, t_{u,s}\right) = \mu_{u,s}\nu_{u,s}\left(k_{u,s}x_{u,s} - \nu_{u,s}{}^{2}t_{u,s}{}^{2}\right) \tag{46}$$

suppressing, according to Scale Relativity, the dynamics with a constant force of fractal type.

As such, the non-stationary Schrödinger equation of fractal type in both scales space and the usual space, in its "universal" instance given through (27), can lead to a wide range of interpretations. The "fractal object"/ particle is in a uniformly accelerated motion of fractal type (see both the argument of the Airy function of fractal type and the expression of the specific potential of fractal type). This, evidently, agrees with the condition of accepting the functionality of de Broglie theory of fractal type, linked to "wave phenomena named fractal object". Generation of probability densities is given by the square of the Airy function of fractal type. As this cannot be integrated on the entire real straight line, the state/ wave package of fractal type can have a center, achievable in the sense of de Broglie theory of fractal type. Then, the state/ wave package of fractal type represents an ensemble of fractal objects/ particles, all showing a uniform rectilinear motion, but each with a different velocity, whereas the argument of the Airy function of fractal type represents a caustic in the dynamics space (the envelope of the ensemble of geodesics which represents the corresponding trajectories). Such an interpretation is related to the nature of the invention of the Airy function: the behavior of light in the proximity of caustics. The functionality of an equivalence principle of fractal type implies that the Airy state/ wave package of fractal type will not scatter, because it represents a fractal object/ particle in an enclosure analogous to Einstein's Elevator, the uniform field of gravitational forces being thus suppressed.

A POSSIBLE MALDACENA TYPE CONJECTURE THROUGH A FRACTAL PARADIGM OF MOTION AND ITS IMPLICATIONS

The in-phase coherence of the dynamics for any physical system implies, through (36b), correlations between two temporal cubics, one specific to the usual space:

$$\Phi(x_u = const, t_u) = \nu_u t_u\left(k_u x_u - \frac{2}{3}\nu_u^2 t_u^2\right) = const \tag{47a}$$

and the other specific to the scales space:

$$\Phi(x_s = const, t_s) = \nu_s t_s\left(k_s x_s - \frac{2}{3}\nu_s^2 t_s^2\right) = const \tag{47b}$$

If the temporal cubics (47a) and (47b) have real roots in the compact forms:

$$
\begin{aligned}
t_{u,s}^{1} &= \frac{h_{u,s}+\bar{h}_{u,s}k_{u,s}}{1+k_{u,s}},\\
t_{u,s}^{2} &= \frac{h_{u,s}+\varepsilon\bar{h}_{u,s}k_{u,s}}{1+\varepsilon k_{u,s}},\\
t_{u,s}^{3} &= \frac{h_{u,s}+\varepsilon^{2}\bar{h}_{u,s}k_{u,s}}{1+\varepsilon^{2}k_{u,s}}
\end{aligned}
\tag{48a}
$$

with $h_{u,s}$, $\bar{h}_{u,s}$ the roots of Hessians and $\varepsilon \equiv \left(-1+i\sqrt{3}\right)/2$ the cubic root of unity $\left(i=\sqrt{-1}\right)$, the values of variables $h_{u,s}$, $\bar{h}_{u,s}$ and $k_{u,s}$ can be "scanned" by simple transitive groups with real parameters. These groups can be revealed through Riemannian spaces associated with the previous cubics. The basis of approach is that the simply transitive groups with real parameters:

$$
x_{u,s}^{l} \leftrightarrow \frac{a_{u,s}x_{u,s}^{l}+b_{u,s}}{c_{u,s}x_{u,s}^{l}+d_{u,s}}\ ,l=1,2,3\ \ a_{u,s},b_{u,s},c_{u,s},d_{u,s}\in R \tag{48b}
$$

where $x_{u,s}^{k}$ are the roots of the cubics (47a) and (47b), induced by $h_{u,s}$, $\bar{h}_{u,s}$ and $k_{u,s}$, whose actions are:

$$
\begin{aligned}
h_{u,s} &\leftrightarrow \frac{a_{u,s}h_{u,s}+b_{u,s}}{c_{u,s}h_{u,s}+d_{u,s}},\\
\bar{h}_{u,s} &\leftrightarrow \frac{a_{u,s}\bar{h}_{u,s}+b_{u,s}}{c_{u,s}\bar{h}_{u,s}+d_{u,s}},\\
k_{u,s} &\leftrightarrow \frac{c_{u,s}\bar{h}_{u,s}+d_{u,s}}{c_{u,s}h_{u,s}+d_{u,s}}k_{u,s}
\end{aligned}
\tag{48c}
$$

The structures of these groups are typical for SL (2R), *i.e.*:

$$
\begin{aligned}
\left[B_{u,s}^{1},B_{u,s}^{2}\right] &= B_{u,s}^{1},\\
\left[B_{u,s}^{2},B_{u,s}^{3}\right] &= B_{u,s}^{3},\\
\left[B_{u,s}^{3},B_{u,s}^{1}\right] &= -2B_{u,s}^{2}
\end{aligned}
\tag{48d}
$$

where $B^{l}_{u,s}$ are the infinitezimal generators of the groups:

$$B^1_{u,s} = \frac{\partial}{\partial h_{u,s}} + \frac{\partial}{\partial \bar{h}_{u,s}},$$

$$B^2_{u,s} = h_{u,s}\frac{\partial}{\partial h_{u,s}} + \bar{h}_{u,s}\frac{\partial}{\partial \bar{h}_{u,s}},$$

(48e)

$$B^3_{u,s} = {h_{u,s}}^2\frac{\partial}{\partial h_{u,s}} + {\bar{h}_{u,s}}^2\frac{\partial}{\partial \bar{h}_{u,s}} + \left(h_{u,s} - \bar{h}_{u,s}\right)k_{u,s}\frac{\partial}{\partial k_{u,s}}$$

It is observed that, in the scales space, the SL(2R) group is assimilated to renormalization groups [10,11], which admit the absolute invariant differentials:

$$\omega^1_{u,s} = \frac{dh_{u,s}}{\left(h_{u,s} - \bar{h}_{u,s}\right)k_{u,s}},$$

$$\omega^2_{u,s} = -i\left(\frac{dk_{u,s}}{k_{u,s}} - \frac{dh_{u,s} + d\bar{h}_{u,s}}{h_{u,s} - \bar{h}_{u,s}}\right), \tag{48f}$$

$$\omega^3_{u,s} = -\frac{k_{u,s}d\bar{h}_{u,s}}{h_{u,s} - \bar{h}_{u,s}}$$

and the 2-forms (the metrics):

$$ds^2_{u,s} = \left(\frac{dk_{u,s}}{k_{u,s}} - \frac{dh_{u,s} + d\bar{h}_{u,s}}{h_{u,s} - \bar{h}_{u,s}}\right)^2 - 4\frac{dh_{u,s}d\bar{h}_{u,s}}{\left(h_{u,s} - \bar{h}_{u,s}\right)^2} \tag{48g}$$

In real terms:

$$h_{u,s} = u_{u,s} + iv_{u,s}, \bar{h}_{u,s} = u_{u,s} + iv_{u,s}, k_{u,s} = e^{i\theta_{u,s}} \tag{48h}$$

and for:

$$\Omega^1_{u,s} = \omega^2_{u,s} = d\theta_{u,s} + \frac{du_{u,s}}{v_{u,s}},$$

$$\Omega^2_{u,s} = \cos\theta_{u,s}\frac{du_{u,s}}{v_{u,s}} + \sin\theta_{u,s}\frac{dv_{u,s}}{v_{u,s}},$$

(48i)

$$\Omega_{u,s}^{3}=-\sin\theta_{u,s}\frac{du_{u,s}}{v_{u,s}}+\cos\theta_{u,s}\frac{dv_{u,s}}{v_{u,s}}$$

the connections with Poincaré representations of the Lobachevsky planes can be obtained. Indeed, the metrics represent a three-dimensional Lorentz structure:

$$ds_{u,s}^{2}=-\left(\Omega_{u,s}^{1}\right)^{2}+\left(\Omega_{u,s}^{2}\right)^{2}+\left(\Omega_{u,s}^{3}\right)^{2}=-\left(d\theta_{u,s}+\frac{du_{u,s}}{v_{u,s}}\right)^{2}+\frac{\mathrm{d}u_{u,s}^{2}+\mathrm{d}v_{u,s}^{2}}{v_{u,s}^{2}} \tag{48j}$$

These metrics reduce to that of Poincaré in the case where $\Omega^{1}\equiv 0$, and define variables $\theta_{u,s}$ as "angles of parallelism" of the hyperbolic planes (the connections). In fact, recalling that:

$$\frac{dk_{u,s}}{k_{u,s}}-\frac{dh_{u,s}+d\bar{h}_{u,s}}{h_{u,s}-\bar{h}_{u,s}}=0\leftrightarrow d\theta_{u,s}=-\frac{du_{u,s}}{v_{u,s}} \tag{48k}$$

represents the connection forms of the hyperbolic planes, relation (48i) will then represent the general Bäcklund transformations in those planes. In such a conjecture it is noted that, if the temporal cubics are assumed to have distinct roots, conditions (48k) are satisfied if and only if the differential forms $\Omega_{u,s}^{1}$ are nules.

Therefore, for metrics (48j) with restrictions (48k), *i.e.*:

$$ds_{u,s}^{2}=\frac{dh_{u,s}d\bar{h}_{u,s}}{\left(h_{u,s}-\bar{h}_{u,s}\right)^{2}}=\frac{\mathrm{d}u_{u,s}^{2}+\mathrm{d}v_{u,s}^{2}}{v_{u,s}^{2}} \tag{49}$$

the parallels transport of the hyperbolic planes actually represents the apolar transports of the temporal cubics.

In the following, indexation *u*, *v* will be left aside. In such a context, metric (48g) offers a natural posibility of extension of the harmonic principle that we associate with a Kepler type problem in a fractal space, by a functional which in fact can represent, for example, the deformation of the nuclear matter in the general case, not only the theory of gravitation in Ernst sense [21,22]. The extension amounts to expressing the energy of mapping by the functional:

$$E(\Phi)=\iiint\left\{\left(\frac{\nabla k}{k}-\frac{\nabla h+\nabla\bar{h}}{h-\bar{h}}\right)^{2}-4\frac{(\nabla h\cdot\nabla\bar{h})}{(h-h^{*})^{2}}\right\}(d^{3}\boldsymbol{x}) \tag{50}$$

The variational principle would then express this simple fact: the most general deformation of the nuclear matter is replicated in the atomic structure by the variation of eccentricity of the electronic orbits. Likewise, inside the nucleus, variation is replicated by an ensemble of harmonic oscillators of the same frequency. The second term from this functional [22] is analogous to the baryonic term from the theory of Skyrme, with the only difference that, now, the whole functional is homogeneous. In the language of skyrmion technology, however [23], it still represents hyperbolic skyrmions, because the geometrical character of the problem is dictated by the metric from (48g).

In this form, the theory illustrates the strong ties of the model of nuclear matter with the ideas of confinement of matter in general. To demonstrate this, we propose a solution to the variational principle related to the energy functional from equation (50). In order to better understand intuitively such a solution, let us start with a 'strange' classical dynamics, the one related to the forces involved in the problem of confinement of the classical ideal gas [22], of inversely proportional magnitude with the distance between molecules. This force is also involved in a Newtonian type description of the inertia [24] which, again, should appear as quite natural in view of the common background of the general relativity and Skyrme theory [25–27].

Assuming a Newtonian type dynamics for this force, the equations of motion can be written as:

$$\ddot{\boldsymbol{r}} + \frac{\mu^2}{r^2}\boldsymbol{r} = 0 \tag{51}$$

This is a plane motion, because the force is central. In polar coordinates of the plane of motion, the equation of motion splits into a system of two differential equations:

$$\ddot{r} - r\dot{\theta}^2 + \frac{\mu}{r} = 0;\ \ r\ddot{\theta} + 2\dot{r}\ddot{\theta} = 0 \tag{52}$$

The second of these gives the area constant:

$$r\dot{\theta}^2 = \dot{a} \tag{53}$$

The first equation (52) can then be integrated as follows [28], but first the time variable needs to be changed into:

$$\tau(t) = \int \frac{dt}{r^2} \therefore \theta'(\tau) = \dot{a} \tag{54}$$

where the prime represents differentiation with respect to τ. Now, if we change the dependent variable into $\xi(\tau) = 1/r$, the first equation (52) becomes:

$$\xi'' + (a^2 - \mu^2)\xi = 0;\ \xi(\tau) \equiv \frac{1}{r} \tag{55}$$

This is the equation of a harmonic oscillator, with the frequency dictated by the rate of area, leading to either trigonometric or hyperbolic functions. We shall consider only the first case, when the solutions take the form:

$$\xi(\tau) = A\sin(\Omega\tau) + B\cos(\Omega\tau),\ \ \Omega^2 \equiv \dot{a}^2 - \mu^2 > 0 \tag{56}$$

Now we can find the relation between the Newtonian time and the new time τ. Indeed, taking equation (56) into equation (54) will give:

$$t \equiv \int \frac{d\tau}{\xi^2(\tau)} = \int \frac{d\tau}{[A\sin(\Omega\tau) + B\cos(\Omega\tau)]^2} \tag{57}$$

and this integral will lead to [29]:

$$\Omega\tau = \phi_0 + \tan^{-1}[(A^2 + B^2)\Omega t] \tag{58}$$

where ϕ_0 is a constant of integration. Interestingly enough, equation (56) represents a particular Kepler motion, corresponding to the null gravitational constant, or null mass, or even null charge, as it were. It cannot be therefore interpreted in terms of the motion of a material point around another attracting material point. However, it can be interpreted in terms of an abstract kinematics, suggested by the analogy between equation (58) and the solution of a Schwartzian type equation [30], which can be considered as the eigenvalue of a stress matrix.

Let us consider the kinematics generated by the differential 1-forms (48i), written as:

$$\begin{aligned} \Omega^1 &= d\phi + \frac{du}{v} \\ \Omega^2 &= \cos\phi\frac{du}{v} + \sin\phi\frac{dv}{v} \\ \Omega^3 &= -\sin\phi\frac{du}{v} + \cos\phi\frac{dv}{v} \end{aligned} \tag{59a}$$

where $\theta \equiv \phi$. In these terms, metric (48g) assumes the standard Lorentzian form (48i). Along the geodesics of this metric, the rates represented by the differential forms (59a) are constant, so that we can find these geodesics from some differential equations involving a parameter linear in the arclength from equation (48g), namely:

$$\begin{aligned} d\phi + \frac{du}{v} &= a \cdot dt \\ \cos\phi\frac{du}{v} + \sin\phi\frac{dv}{v} &= b \cdot dt \\ -\sin\phi\frac{du}{v} + \cos\phi\frac{dv}{v} &= c \cdot dt \end{aligned} \tag{59b}$$

with a, b and c some constants. The last two equations give:

$$\frac{\dot{u}}{v} = b\cos\phi - c\sin\phi \tag{60}$$

$$\frac{\dot{v}}{v} = b\sin\phi + c\cos\phi$$

and then, from the first of (59b), we have:

$$\dot{\phi} = a - b\cos\phi + c\sin\phi \tag{61}$$

an equation which can be immediately integrated. We prefer to perform this integration by putting the right hand side of (61) in the form of a perfect square, in order to show that this corresponds to the equation (57) above. Indeed, if we take $2\Omega\tau \equiv \phi$, we can write the integral in the form (58) for a, b, c given by:

$$\begin{aligned} a &\equiv \frac{A^2 + B^2}{2} \\ b &\equiv \frac{A^2 - B^2}{2} \\ c &\equiv AB \end{aligned} \tag{62}$$

This gives an interpretation to the classical “time” variable τ, provided we know something about this abstract kinematics. Insofar as the parameters u and v are concerned, using equation (60) we obtain:

$$\frac{v}{v_0} = a - b\cos\phi + c\sin\phi \tag{63}$$

where v_0 is another integration constant. Therefore, v represents the inverse square of the position vector of the previously described motion. On the other hand, for parameter u, we find the following solution:

$$v_0(u - u_0) = b\sin\phi + c\cos\phi \tag{64}$$

where u_0 is another constant of integration.

Now, one can find a particular solution to the variational principle applied to the energy functional (50) along the geodesics given by equations (63) and (64), if assuming that their parameter (and therefore phase ϕ) is a solution of the Laplace equation. This can be easily proved by continuing to work with real parameters u, v and ϕ as before. The Euler-Lagrange equations associated with the variational principle applied to (48g) are:

$$\begin{aligned} &\nabla \cdot \left(\nabla\phi + \frac{\nabla u}{v}\right) = 0 \\ &\nabla^2\phi - \nabla\phi \cdot \frac{\nabla v}{v} = 0 \\ &\nabla^2(\ln v) - \nabla\phi \cdot \frac{\nabla u}{v} = 0 \end{aligned} \tag{65}$$

On the other hand, the geodesics of the metric (48g) are solutions of the system of differential equations:

$$\left(\frac{u'}{v}\right)' + \phi' \cdot \frac{v'}{v} = 0$$
$$\phi'' - \phi' \cdot \frac{v'}{v} = 0 \tag{66}$$
$$(\ln v)'' - \phi' \cdot \frac{u'}{v} = 0$$

where the prime means differentiation with respect to the parameter of geodesics. Now, if functions u, v and ϕ depend on position only through the parameter of geodesics, equations (65) can be written as:

$$\left(\phi' + \frac{u'}{v}\right)' (\nabla t)^2 + \left(\phi' + \frac{u'}{v}\right)\nabla^2 t = 0$$
$$\left(\phi'' - \phi' \cdot \frac{v'}{v}\right)(\nabla t)^2 + \phi' \nabla^2 t = 0 \tag{67}$$
$$\left((\ln v)'' - \phi' \cdot \frac{u'}{v}\right)(\nabla t)^2 + (\ln v)' \nabla^2 t = 0$$

The first terms of these equations are zero, as a consequence of the equations of geodesics. So, we have still another form of the harmonic principle: under fairly general conditions, the harmonic mapping corresponding to energy (50) is given by the geodesics of the metric, provided their parameter is a regular harmonic function. This of course makes the phase ϕ the arctangent of such a function, according to equation (61) above.

Therefore, at least in this particular instance, we have to focus on the geodesics of metric (48g). The Killing vectors of the metric represent conservation laws, and thus parameters a, b, c from equations (63) and (64) represent the expression of these conservation laws. Equations (63) and (64) themselves represent two Kepler motions of fractal type, and by this we are certainly in position to know exactly the field of application of the above kinematics: it is the theory of space stresses, involved in the fractal Kepler type problem. These stresses are induced in the core of the solar system, for instance, or in the nucleus, and they represent a material point – in Hertz sense – whose particles, acted upon by the inverses of the space elastic forces, behave like an ideal gas. The density of this material point varies inversely proportional with the square of distance from the origin of the reference frame, which is actually its origin. We have to insist upon these two aspects of the problem of action at distance, for they are instrumental in understanding how the nucleus works.

As unanimously known, one of the first theories on nuclear matter was that of a gas model [31]. Even if it was not so successful, it certainly approached a fundamental aspect of the structure of nuclear matter, which, actually, turns out to be the fundamental problem regarding the structure of the matter in general. In view of the above discussion, we may assume the following scenario: the particles of nucleus are decided "in pairs" by the inversion transformation between the external Newtonian forces and the confinement internal to the nucleus. The last ones are represented as harmonic oscillators – gluons as it were – and described by a general dynamics. The kinematics of a pair is represented as two fractal Kepler motions given by equations (63) and (64) above. Here, of course, we assume that the pair is 'accidentally decided' by inversion but, once decided, it is described by an actual state of stress whose kinematics can be "classically" described. This state of stress can be physically characterized by a statistics of a type specifically connected with light [22].

With equations (63) and (64) above, we certainly can extend to matter, particularly to the nucleus, Mac Cullagh's view about the structure of light [32]. Indeed, the two equations represent two harmonic oscillators, as well as two fractal Kepler type motions. In "classical" terms, one can say that two particles (partons) inside the nucleus have independent motions of Kepler type. Even though classically described, such an image should be statistically accessible to measurement through some kind of stresses or strains, and reflected in the eccentricity of electronic orbits.

This image of the nuclear structure can become even more precise, from the very classical point of view we advocate in this work. Indeed, in as much as we are thinking in relation with the Newtonian natural philosophy, the above view on Hertz's particles – the partons of nuclear matter – recovers, and solves, we should say, in a "quantum-mechanical" way, one of the most important issues of such philosophy, left behind by Newton in his theory of forces.

CONCLUSIONS

A Maldacena type conjecture in relation with Scale Relativity Theory using Schrödinger representations of fractal type dynamics of any physical system is proposed. "Holographic implementations" of the dynamics for any physical system in the usual space are defined and discussed. "Holographic implementations" of the dynamics for any physical system in the scales space are developed. The dual "holographic implementations" through in-phase coherences of the dynamics for any physical system are shown to be operational. Through such a mechanism, the apolar transport of the temporal cubics of fractal type is proven as operational, as well. In the Riemannian spaces associated with the cubics, the apolar transport appears equivalent to the parallel transport of the hyperbolic planes. It is shown that any dynamics which reflects SL (2R) "processes" has intrinsic hyperbolic geometries. The harmonic mappings from usual and scales spaces to hyperbolic ones involve correlations between General Relativity in the sense of Ernst and Skyrme Theory.

R E F E R E N C E S

1. MALDACENA J., *The Large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys., 1998, **2**, 231.
2. GUBSER S.S., KLEBANOV I.R., POLYAKOV A.M., *Gauge theory correlators from noncritical string theory*, Phys. Lett., 1998, **105**, B428.
3. Witten E., *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys., 1998, **2**, 253.
4. BAR-YAM, Y., *Dynamics of Complex Systems, The Advanced Book Program*, Addison-Wesley, Reading, Massachusetts, USA. 1997.
5. MITCHELL, M., *Complexity: A Guided Tour*, Oxford University Press, Oxford, UK, 2009.
6. BADII, R., *Complexity: Hierarchical Structures and Scaling in Physics*, Cambridge University Press, UK, 1997.
7. FLAKE, G.W., *The Computational Beauty of Nature*, MIT Press, Cambridge, MA, UK, 1998.
8. BĂCEANU, D. DIETHELM, K. SCALAS, E. TRUJILLO, H., *Fractional Calculus, Models and Numerical Methods*, Singapore, World Scientific, 2016.
9. ORTIGUERIA, M.D., *Fractional Calculus for Scientists and Engineers*, Springer, 2011.
10. NOTTALE, L., *Scale Relativity and Fractal Space-Time: A New Approach to Unifying Relativity and Quantum Mechanics,* Imperial College Press, London, UK, 2011.
11. MANDELBROT B. B., *The Fractal Geometry of Nature*, W. H. Freeman and Co., San Francisco, USA, 1982.
12. MERCHES, I., Agop, M., *Differentiability and fractality in dynamics of physical systems*, World Scientific, New Jersey, USA, 2016.
13. AGOP, M., PAUN, V.P., *On the new perspectives of the fractal theory. Fundaments and applications*, Romanian Academy Publishing House, Bucharest, Romania, 2017.
14. JACKSON, E.A., *Perspective of nonlinear Dynamics*, Vol. 2, Cambridge University Press, Cambridge, UK, 1991.
15. CRISTESCU, C.P., *Nonlinear dynamics and chaos. Theoretical fundaments and applications*, Romanian Academy Publishing House, Bucharest, Romania, 2008.
16. AGOP, M., NICA, P.E., GURLUI, S., FOCSA, C., PAUN. V.P., COLOTIN, M., *Implications of an extended fractal hydrodynamic model*, The European Physical Journal, 2010, **56**, 405-419.
17. AGOP, M., PAUN, V.P., HARABAGIU, A., *El Naschie's ε(∞) theory and effects of nanoparticle clustering on the heat transport in nanofluids*, Chaos Sol. Fract., 2008, **37** (*5*), 1269-1278.
18. AGOP, M., NICA, P.E., IOANNOU, P.D., ANTICI, A., PAUN, V.P., *Fractal model of the atom and some properties of the matter through an extended model of scale relativity,* The Euro. Phys. J. D., 2008, **49** (*2*), 239-248.
19. AGOP, M., MURGULET, C., *El Naschie's ε(∞) space-time and scale relativity theory in the topological dimension D=4*, Chaos Sol. Fract., 2007, **32** (*3*), 1231-1240.
20. GURLUI, S., AGOP, M., STRAT, M., STRAT, G., BACAITA, S., CEREPANIUC, A., *Some experimental and theoretical results on the anodic patterns in plasma discharge*, Phys. Plasmas, 2006, **13** (*6*), 063503-06350310.
21. MAZILU, N., AGOP, M., *At the Crossroads of Theories. Between Newton and Einstein - The Barbilian Universe (in Romanian);* ArsLonga Publishing House, Iasi, Romania, 2010.
22. MAZILU, N. AGOP, M., *Skyrmions: A Great Finishing Touch to Classical Newtonian Philosophy*; World Philosophy Series; Nova: New York, USA, 2012.

23. ATIYAH, M. F., SUTCLIFFE, P., The Geometry of Point Particles, *Proc. Roy. Soc. Lond*., 2002, **A458**, 1089-1116.
24. SCIAMA, D. W., *The Physical Foundations of General Relativity*, Doubleday, New York, SUA, 1969.
25. SKYRME, T.H.R., A Non-Linear Field Theory, *Proc. Roy. Soc. Lond*., 1961, **A260**, 127-138.
26. SKYRME, T.H.R., The Origins of Skyrmions, *Int. J. Modern Phys*., 1988, **A3**, 2745-2751.
27. SKYRME, T.H.R., *Selected Papers, with Comentary*, G.E. Brown (Ed.), World Scientific Series in 20th Century Physics, Vol. 3, 1994.
28. GOVINDER, K. S., ATHORNE, C. LEACH, P. G. L., The Algebraic Structure of Generalized Ermakov Systems in Three Dimensions, *J. Phys. A: Math. Gen*., 1993, **26**, 4035-4046.
29. GRADSHTEYN, I. S., RYZHIK, I. M., *Table of Integrals, Series and Products*, Fifth Edition, A. Jeffrey (Ed.), Academic Press, 1994.
30. MIHAILEANU, N., *Complements of Geometry: Analytical, Projective and Differential*, Pedagogical Publishing House, Bucharest, Romania, 1972.
31. FERMI, E., *High Energy Nuclear Events*, Progress Th. Phys., *(*Japan), 1950, **5**, 570-583.
32. MAC CULLAGH J., On the Intensity of Light when the Vibrations are Elliptical, *Edinburgh Journal of Science*, **1831**, 86-88.