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MATHEMATICS

A RICCATI EQUATION APPROACH TO CONSTRUCT NEW DISPERSIVE SOLITONS TO THE KAUP-NEWELL EQUATION

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The present paper examines analytically the Kaup-Newell equation, an important class of nonlinear Schrodinger equations with lots of applications in optical fibers. The Riccati equation approach is employed for the present study. The method is one of the recent active integration methods that transform the given partial differential equation to an ordinary one, and thereafter to algebraic equations. Various new optical soliton solutions are successfully disclosed by the method, including bright-singular, dark-singular, and singular optical soliton solutions to mention a few. Similarly, the constraint conditions involved are determined in relation to each soliton solution and illustrate some of the obtained solutions graphically. Finally, the present study can help in understanding many phenomena in optics in particular, and in mathematical physics in general.

Keywords: Riccati equation method, optical solitons, Kaup-Newell equation, Nonlinear Schrodinger equation

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1. INTRODUCTION

Nonlinear Schrodinger equations (NLSE) are evolution equations that play vital roles in different branches of modern science and technology, with the Kaup-Newell Equation (KNE) [1–11] as an important member with many applications in optical fibers, among others. The mechanics of solitons in optical devices is considered to be a vital field of investigation in the present day communication industries, and has gained much interest in the past few decades [12–20]. The propagation of waves in optical devices with Kerr dispersion remains essential in connection to the so-called evolution equations [21–25]. More, a famous class of equations that originate from NLSE is represented the Derivative Nonlinear Schrodinger Equations (DNLSE). This class is further divided into I, II and III, respectively. In particular, the DNLSE-I is also known as the KNE. This equation, which has been thoroughly investigated mostly numerically in the literature, will be the focal point of the present study. Furthermore, many integration schemes and numerical techniques were employed in the last decade to examine different types of both the real and complex time evolution equations models including those in references cited above together, with some in [26–35] and the references therein among other.

However, the present paper will examine the KNE with Kerr nonlinearity [1–11] analytically using the Riccati equation approach [18–20]. The Riccati equation method or approach is an analytical method that starts off by transforming the given KNE to an ordinary differential equation which will then be solved simultaneously after obtaining a system of algebraic equations. Also, a variety of new hyperbolic optical soliton solutions are set to be disclosed using the method which will then be illustrated graphically. Further, the arrangement of the current manuscript takes the follows order: Section 2 gives some basics information on the KNE, while Section 3 gives some preliminaries and method of solution. Section 4 determines the exact solitons *via* the Riccati equation procedure for the KNE, and Sections 5 and 6 present the graphical illustrations and conclusion, respectively.

2. GOVERNING EQUATION

The Kaup-Newell model is a class of nonlinear Schrodinger differential equation is given in the dimensionless form as [1–6]:

$$iq_t + aq_{xx} + ib(|q|^2 q)_x = ic(|q|^2)_x \ q, \tag{1}$$

where $q=q(x, t)$ is the profile of the wave characterized by a complex-valued function and relies on the temporal and spatial variables, t and x, respectively. Further, the terms on the LHS of the equation denote the evolution, Group Velocity Dispersion (GVD) and non-Kerr dispersion term. Also, constants a and b are the coefficients of the GVD and the self-steepening term, respectively. Furthermore, the RHS term is representing a nonlinear dispersion with real constant coefficient parameter c. More on the KNE including its steeping nonlinearity ability, phase modulation, as well as its various applications in optical fibers, can be found in [7–11] and the references therein.

3. PRELIMINARIES

The solution of (1) starts off by assuming the wave profile of the following form [20–21, 33–34]:

$$q(x,t) = u(\eta)e^{i\phi(x,t)}, \tag{2}$$

with wave's amplitude $u(\eta)$, where $\eta = x - \gamma t$. Also, $\phi(x,t)$ is the wave's phase component prescribed as follows:

$$\phi(x,t) = -kx + \omega\, t + \theta_0, \tag{3}$$

where θ_0 denotes the phase constant, ω represents the frequency, and k stands for the wave number which are all associated with the soliton.

Further, making use of (2) in (1), the subsequent set of partial differential equations from the corresponding real and imaginary parts is respectively acquired as:

$$au_{xx} - (ak^2 + \omega\,)u + bk\, u^3 = 0, \tag{4}$$

and

$$-\gamma\, u_t - 2aku_x + (3b - 2c)u^2 u_x = 0, \tag{5}$$

Also, equation (5) gives the following constraint condition; see [1-11]

$$3b - 2c = 0, \tag{6}$$

and consequently, the speed of the soliton v is as follows:

$$v = \frac{2ak-\alpha}{\gamma},\ \ \gamma \neq\ 0. \tag{7}$$

With the aid of equation (6), (4) reduces to:

$$a\, u_{xx} - (\omega + ak^2)u + \frac{2}{3}\, cku^3 = 0. \tag{8}$$

Now, to solve the above equation given in (8), we make use of the transformation that governs the movement of the wave, as follows:

$$u = u(\eta),\ \ \eta = B(-v\, t + x), \tag{9}$$

and thus obtain from equation (8) the equation below:

$$a\, B^2\, u_{\eta\eta} - (\omega + ak^2)u + \frac{2}{3}\, cku^3 = 0, \tag{10}$$

or equivalently expressed as:

$$n_1\, u_{\eta\,\eta} + n_2\, u + n_3\, u^3 = 0, \tag{11}$$

with

$$\begin{cases} n_1 = a\, B^2, \\ n_2 = -(\omega + ak^2), \\ n_3 = \frac{2}{3}\, ck\,. \end{cases} \tag{12}$$

Equation (11) is a Nonlinear Ordinary Differential Equation (NODE) that will be examined in the subsequent section by virtue of the Riccati equation method for the complete analysis of the KNE.

4. APPLICATION TO KAUP-NEWELL MODEL

The present section demonstrates the relevance of the Riccati equation method [18–20] in revealing variety optical solitary wave solutions to the KNE. In doing so, new and more general optical soliton solutions will be revealed.

The method first goes by letting the solution of equation (11) to be of the form:

$$u(\eta) = a_0 + \sum_{j=1}^{n} a_j \phi^{j}(\eta) + \sum_{j=1}^{n} b_j \phi^{-j}(\eta), \tag{13}$$

and then obtain terms the $u_{\eta\eta}$ and u^3 from equation (13), n to be determined by the concept of homogenous balancing method, [29-30]. Further, $\phi(\eta)$ must satisfy the Riccati differential equation given, as follows:

$$\phi'(\eta) = P\phi^2(\eta) + Q\phi(\eta) + R. \tag{14}$$

Now, with the homogenous balancing method, n is found, that is $n = 1$; which reduces equation (13) to:

$$u(\eta) = a_0 + a_1\phi(\eta) + b_1\phi^{-1}(\eta). \tag{15}$$

Substituting equation (15) into (11) results into a system of algebraic equations given, as follows:

$$\begin{cases} 2\,a_1 n_1 P^2 + a_1^3 n_3 = 0, \qquad 2\,b_1 n_1 R^2 + b_1^3 n_3 = 0, \\ 3\,a_0 b_1^2 n_3 + 3\,b_1 n_1 Q\,R = 0,\ 3\,a_1 n_1 P\,Q + 3\,a_0 a_1^2 n_3 = 0, \\ 6\,a_1 a_0 b_1 n_3 + a_1 n_1 Q\,R + a_0^3 n_3 + a_0 n_2 + b_1 n_1 P\,Q = 0, \\ 3\,a_1 b_1^2 n_3 + 3\,a_0^2 b_1 n_3 + 2\,b_1 n_1 P\,R + b_1 n_1 Q^2 + b_1 n_2 = 0, \\ 3\,a_1^2 b_1 n_3 + 2\,a_1 n_1 P\,R + a_1 n_1 Q^2 + a_1 n_2 + 3\,a_0^2 a_1 n_3 = 0 \end{cases} \tag{16}$$

The set of equations given above reveal the values of the constants a_0, a_1 and b_1, as follows:

$$\begin{cases} a_0 = -\frac{i\sqrt{n_1}\,Q}{\sqrt{2B}}, \\ b_1 = -\frac{i\sqrt{2n_1}\,R}{\sqrt{B}}, \\ a_1 = \frac{i\left(\left(n_1\left(Q^2-4\,P\,R\right)-2\,n_2\right)\right)}{6\sqrt{2Bn_1}R} \end{cases}. \tag{17}$$

We therefore determine the solution cases below by finding the function $u(\eta)$, together with the possible situations of $R,\ Q,\ P$, for possible forms of the Riccati equation, while c_1 is any arbitrary constant.

Additionally, we get from the second equation in (12) soliton's frequency ω in terms k; the wave number together with n_2, as follows:

$$\omega = -\left(\frac{n_2 + ak(1+k)}{1+bk}\right), \tag{18}$$

where $bk \neq 1$. Thus, we give the following solution cases for the KNE that are determined via the Riccati equation method, as follows:

Case 1:

In the present case, we obtain:

$$\begin{cases} P = \dfrac{Q^2 - \frac{2n_2}{n_1}}{4R}, \\ \phi(\eta) = \dfrac{\sqrt{4PR-Q^2}\tan\left(\frac{1}{2}\left(c_1\sqrt{4PR-Q^2}+\eta\sqrt{4PR-Q^2}\right)\right)-Q}{2P}, \\ u(\eta) = -\sqrt{\dfrac{1}{n_3}}\left(\dfrac{\sqrt{-2n_1n_2}Q\tanh\left(\frac{\sqrt{n_2}(c_1+\eta)}{\sqrt{2n_1}}\right)+2n_2}{2\sqrt{n_2}\tanh\left(\frac{\sqrt{n_2}(c_1+\eta)}{\sqrt{2n_1}}\right)+\sqrt{2}\sqrt{n_1}Q}\right), \end{cases} \tag{19}$$

which reveals a dark soliton solution given as:

$$q(x,t) = -\sqrt{\frac{1}{n_3}}\left(\frac{\sqrt{-2n_1n_2}Q\tanh\left(\frac{\sqrt{n_2}(c_1+\eta)}{\sqrt{2n_1}}\right)+2n_2}{2\sqrt{n_2}\tanh\left(\frac{\sqrt{n_2}(c_1+\eta)}{\sqrt{2n_1}}\right)+\sqrt{2}\sqrt{n_1}Q}\right)e^{i(-kx+\omega t+\theta)}. \tag{20}$$

However, a singular soliton solution remains a special case of equation (20) which will be given in the proceeding case 4.

Case 2:

The following parameters and functions are obtained in this situation:

$$\begin{cases} P = 0,\ Q = -\dfrac{\sqrt{2n_2}}{\sqrt{n_1}}, \\ \phi(\eta) = c_1 e^{\eta Q} - \dfrac{R}{Q}, \\ u(\eta) = -\sqrt{\dfrac{n_2}{n_3}}\left(1 - \dfrac{4c_1\sqrt{n_2}}{\sqrt{2n_1}Re^{\left(\frac{\sqrt{2n_2}\eta}{\sqrt{n_1}}\right)}+2c_1\sqrt{n_2}}\right), \end{cases} \tag{21}$$

which gives the following bright-singular soliton solution:

$$q(x,t) = -\sqrt{\frac{n_2}{n_3}}\left(1 - \frac{4c_1\sqrt{n_2}}{\sqrt{2n_1}R\left(\cosh\left(\frac{\sqrt{2n_2}\eta}{\sqrt{n_1}}\right) - \sinh\left(\frac{\sqrt{2n_2}\eta}{\sqrt{n_1}}\right)\right) + 2c_1\sqrt{n_2}}\right)e^{i(-kx+\omega t+\theta)}, \quad (22)$$

that is valid only when $n_3 < 0$.

Case 3:

The present solution case gives the following:

$$\begin{cases} P = 0,\ Q = \frac{\sqrt{2n_2}}{\sqrt{n_1}} \\ \phi(\eta) = e^{Q\eta} - \frac{R}{Q}, \\ u(\eta) = \sqrt{\frac{-1}{n_3}}\left(\frac{2R}{R - c_1 Q e^{Q\eta}} - n_2\right). \end{cases} \quad (23)$$

The relations determined in equation (23) collectively posed again a different soliton solution; precisely a bright-singular soliton, given as:

$$q(x,t) = \sqrt{\frac{-1}{n_3}}\left(\frac{2R}{R - c_1\frac{\sqrt{2n_2}}{\sqrt{n_1}}\left(\cosh\left(\frac{\sqrt{2n_2}\eta}{\sqrt{n_1}}\right) + \sinh\left(\frac{\sqrt{2n_2}\eta}{\sqrt{n_1}}\right)\right)} - n_2\right)e^{i(-kx+\omega t+\theta)}, \quad (24)$$

which exists only for $n_1 n_2 < 0$.

Case 4:

The following solution case is obtained to be:

$$\begin{cases} Q = 0,\ P = -\frac{n_2}{2n_1 R}, \\ \phi(\eta) = \frac{\sqrt{R}\tan\left(c_1\sqrt{PR} + \eta\sqrt{PR}\right)}{\sqrt{P}}, \\ u(\eta) = -\sqrt{\frac{-n_2}{n_3}}\coth\left(-\frac{\sqrt{n_2}}{\sqrt{2n_1}}(\eta + c_1)\right), \end{cases} \quad (25)$$

which yields the following solution:

$$q(x,t) = -\sqrt{\frac{-n_2}{n_3}}\coth\left(-\frac{\sqrt{n_2}}{\sqrt{2n_1}}(\eta + c_1)\right)e^{i(-kx+\omega t+\theta)}. \quad (26)$$

The solution given in equation (26) is a singular soliton solution existing with similar constraint condition with case 1, coupled with an additional condition *PR*>0.

Case 5:

In this case, the following information is revealed by the method:

$$\begin{cases} Q = 0,\ P = -\frac{n_2}{8n_1R}, \\ \phi(\eta) = \frac{\sqrt{R}\tan\left(c_1\sqrt{PR}+\eta\sqrt{PR}\right)}{\sqrt{P}}, \\ u(\eta) = -\sqrt{\frac{-n_2}{4n_3}}\left(\coth\left(-\frac{\sqrt{n_2}}{\sqrt{8n_1}}(\eta + c_1)\right) + \tanh\left(-\frac{\sqrt{n_2}}{\sqrt{8n_1}}(\eta + c_1)\right)\right), \end{cases} \tag{27}$$

which then gives the dark-singular soliton solution, as follows:

$$q(x,t) = -\sqrt{\frac{-n_2}{4n_3}}\left\{\coth\left(-\frac{\sqrt{n_2}}{\sqrt{8n_1}}(\eta + c_1)\right) + \tanh\left(-\frac{\sqrt{n_2}}{\sqrt{8n_1}}(\eta + c_1)\right)\right\} e^{i(-kx+\omega t+\theta)} \tag{28}$$

which is indeed valid for similar constraint condition given in case 1, coupled to the RP>0 as additional condition.

Case 6:

We get the following solution case:

$$\begin{cases} Q = 0,\ P = \frac{n_2}{4n_1R}, \\ \phi(\eta) = \frac{\sqrt{R}\tan\left(c_1\sqrt{PR}+\eta\sqrt{PR}\right)}{\sqrt{P}}, \\ u(\eta) = \sqrt{\frac{-2n_2}{n_3}}\csc\left(\frac{\sqrt{n_2}\,(\eta+c_1)}{\sqrt{n_1}}\right), \end{cases} \tag{29}$$

which then gives the following solution:

$$q(x,t) = -\sqrt{\frac{-2n_2}{n_2}}\operatorname{csch}\left(\frac{\sqrt{-n_2}\,(\eta+c_1)}{\sqrt{n_1}}\right) e^{i(-kx+\omega t+\theta)}. \tag{30}$$

More, the solution above is a singular soliton that exists with the constraint conditions $n_2 n_3 > 0, n_1 n_2 < 0$ and *RP*>0.

5. RESULTS AND DISCUSSION

The present study examines analytically the KNE. A procedure called the Riccati equation method [18–20] is employed for the present examination. However, various solutions of the KNE have been given in the past using many integration schemes, including those in [1–10]. Accordingly, the present method disclosed a new set of optical solitons including the bright-singular soliton, singular soliton, and dark-singular soliton solutions, respectively, and many combinations. The presented solutions happened to be the generalization of the solutions recently devised by Jawad *et al.* [11], and are believed to help in understating more about the dynamics of the KNE. Below, we give the three-dimensional plots of certain solutions by fixing some parameters, as stated.

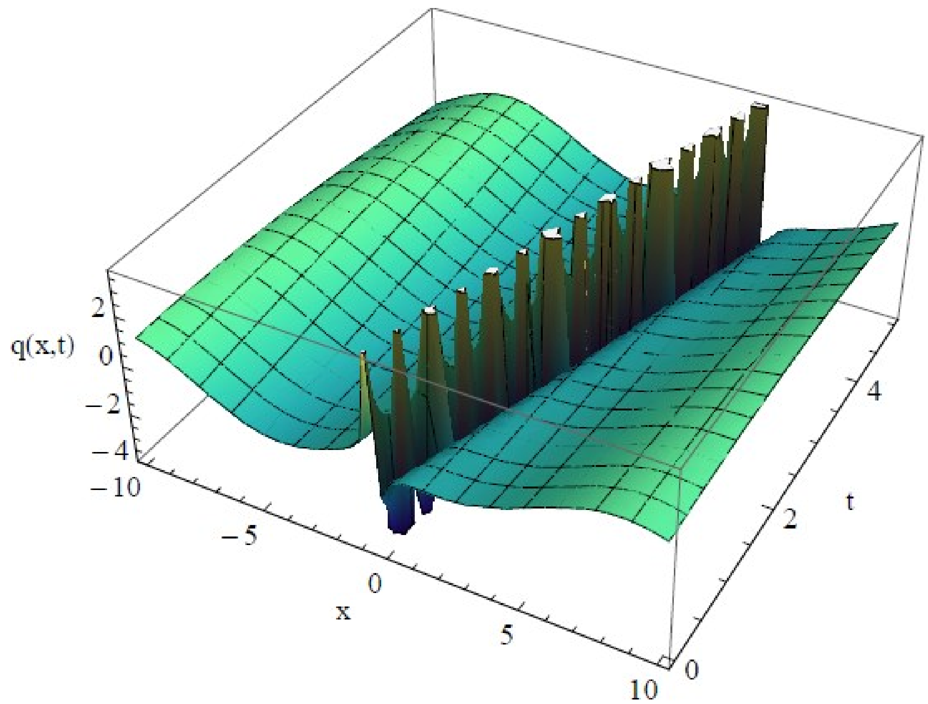

Fig. 1. Graphical depiction of the soliton given in Eq. (20) using $n_1 = 1$, $n_2 = 1$, $n_3 = 1$, $R = 0.1$, $Q = 0.1$, $\gamma = 1$, $c_1 = 0.5$, $k = 0.5$, $\omega = 0.4$, $\theta = 1$.

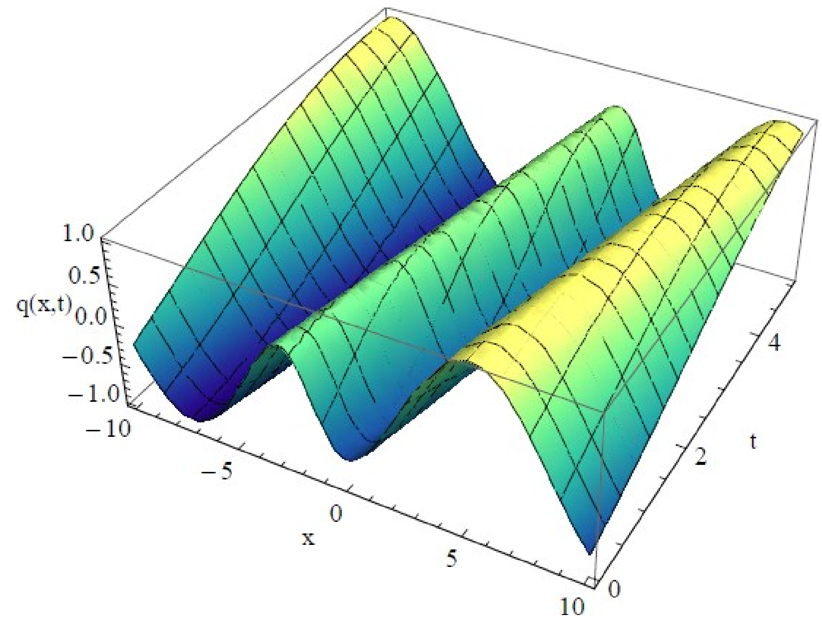

Fig. 2. Graphical depiction of the soliton given in Eq. (22) using
$n_1 = 1$, $n_2 = 1$, $n_3 = 1$, $R = 0.1$, $Q = 0.1$, $\gamma = 1$, $c_1 = 0.5$, $k = 0.5$, $\omega = 0.4$, $\theta = 1$.

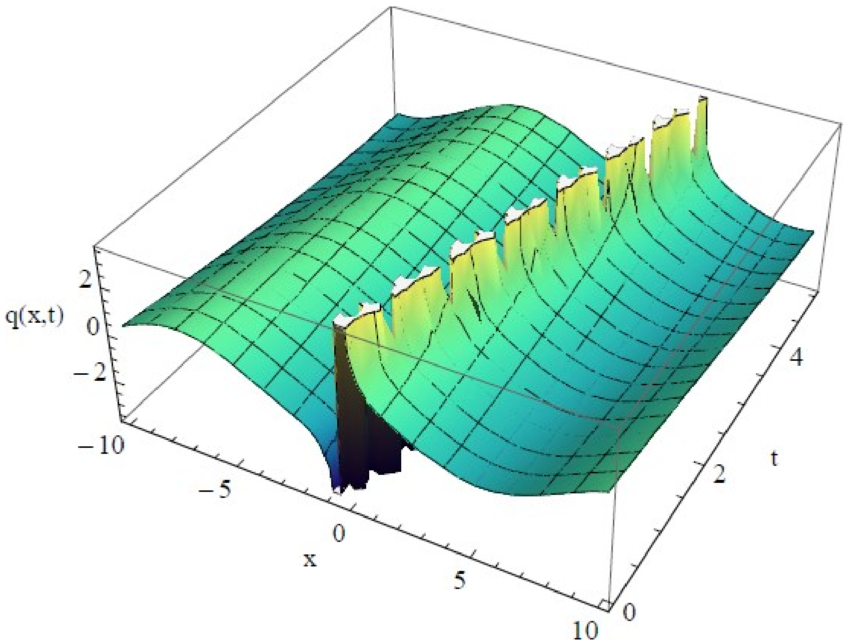

Fig. 3. Graphical depiction of the soliton given in Eq. (26) using
$n_1 = 1$, $n_2 = 1$, $n_3 = 1$, $R = 0.1$, $Q = 0.1$, $\gamma = 1$, $c_1 = 0.5$, $k = 0.5$, $\omega = 0.4$, $\theta = 1$.

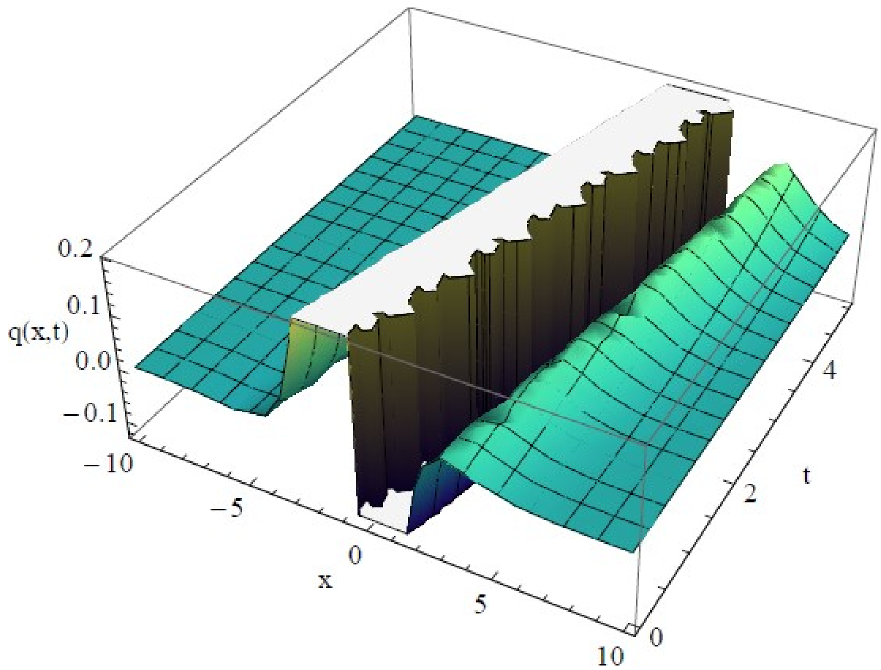

Fig. 4. Graphical depiction of the soliton given in Eq. (30) using
$n_1 = 1$, $n_2 = 1$, $n_3 = 1$, $R = 0.1$, $Q = 0.1$, $\gamma = 1$, $c_1 = 0.5$, $k = 0.5$, $\omega = 0.4$, $\theta = 1$.

6. CONCLUSIONS

The present paper analytically examined the Kaup-Newell equation, being an important class of nonlinear Schrodinger equations which plays vital part in optical fiber transmission, among other applications. A relatively new method, nameby the Riccati equation method, is employed for the present study. The method is one of the recent active integration methods that first transform the given partial differential equation to an ordinary one, and thereafter to algebraic equations which are tackled with the aid of computer algorithms, and then reverse the process to determine the overall general solution. Indeed, the method proved to be successful and disclosed a set of new hyperbolic solutions or rather the optical soliton solutions comprising singular solitons, bright-singular soliton and dark-singular soliton solutions, to mention a few. Thus, the used method is greatly recommended for construction of soliton solutions to various classes of real and complex evolution equations that play vital part in optical fiber and mathematical physics.

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