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MATHEMATICS

PLEASURE, IMAGINATION, FEAR AND JOY: APPLIED THEMES IN NICOLE ORESME'S *DE CONFIGURATIONIBUS*

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The present work explores the idea that concepts present in Nicole Oresme's work, and rooted in the medieval revival of Corpus Aristotelicum, are reflected throughout active areas of research today. Nicole Oresme's volume, titled *Tractatus de configurationibus qualitatum et motuum*, approximately written between 1351 and 1355, showcases early mathematical applications that would presently be classified as works in computational biology and applied mathematics. Oresme uses his "doctrine of configurations" to investigate various real-world phenomena, alluding to several examples that align with recent developments in applied mathematics. In our paper, we investigate Oresme's sources, motivations, examples and models, and their correspondences with recent works in the field of mathematical modeling. We focus on two classes of applications: (1) modeling pleasure and imagination, and (2) modeling subjective perceptions such as fear, hope, sorrow, and joy.

Keywords: mathematical modeling, applied mathematics, Nicole Oresme, 14th century.

1. OUR MOTIVATION

The medieval scholar Nicole Oresme developed his "doctrine of configurations" to understand real world occurrences. In this process, he presents examples of natural

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phenomena that only recently were successfully modeled. Some of his models are surprising, as he speaks of abstract notions, such as joy, pain, pleasure, and sorrow. To highlight the importance of his work, we quote the leading neurologist, Antonio Damasio, who wrote [8] in a volume on Spinoza that "Given the ubiquity of feelings, one would have thought that their science would have been elucidated long ago - what feelings are, how they work, what they mean - but that is hardly the case. Of all the mental phenomena we can describe, feelings and their essential ingredients - pain and pleasure - are the least understood in biological and specifically neurobiological terms." Hence, serious scientific problems are related to the context of ideas we will be exploring in our quest. These themes are present in Oresme's *De configurationibus*. If works detailing the application of Archimedes' philosophy survived, the origin of applied mathematics could be traced to him. Yet, with few of Aristotle's original books available, we are left asking: *when did philosophers begin to explain natural phenomena through mathematics*? This is the main question we bear in mind while we pursue our present analysis.

Common perception holds that medieval sciences contributed little to the modern scientific revolution. Despite this notion, Edward Grant [11] asks: "Even if the Middle Ages made few significant contributions to the advancement of the sciences themselves, or none at all, [...] if no noteworthy medieval contributions were made to help shape specific scientific advances in the seventeenth century, in what ways did the Middle Ages contribute to the Scientific Revolution and, more to the point, lay the foundations for it?" [30]. Richard Rubinstein [25] underlines that: "Two names are sufficient to explode the myth of medieval ignorance: Jean Buridan and Nicole Oresme," and further assesses that "like bold scientific thinkers ever since, they took an existing paradigm (the Aristotelian theory of motion) as their starting point, and ended by revolutionizing it." Our work focuses on philosopher Nicole Oresme, whose texts we survey to find evident parallels with contemporary applied mathematics. Contemplating Oresme's influence on the scientific revolution, Dirk Struik describes Oresme's work as "an exception", in the sense that it "forms a direct link with Renaissance science," [33]. For extensive discussions on his contributions in his historical time, see also [7, 12].

2. NICOLE ORESME'S BIOGRAPHY

Oresme was born around 1320 in the village of Allemagne, near Caen, or present-day Fleury-sur-Orne. Oresme was a "bursar" of the College of Navarre from 1348 to 1356, when he became a Master. Oresme's major was in Theology, but it is not known when he earned his degree of Master in Theology. At the College of Navarre he wrote his most important works, e.g. *De proportionibus proportionum*, which is of particular importance for the history of mathematics, and *Ad pauca respicientes*, of interest for the history of ideas in celestial mechanics.

Oresme remained Master of the College until December 4, 1361, when, during complex political times, he was forced to resign. On November 23, 1362 he became a canon of the Rouen Cathedral, and on March 18, 1364 dean of the Cathedral. Researchers speculate that during this time Oresme served as the King's confessor and adviser, but his political influence is difficult to assess. Before 1370, Oresme acted as one of Charles V's (1364-1380) chaplains. At King's court, Oresme was considered an expert in Aristotle's work; at king's request he translated, from Latin into French, Aristotle's *Ethics*, *Politics*, and *Economics*.

In a related paper, the first two of the present authors discussed Nicole Oresme's anticipation of developments of curvature [30], which speaks plenty about Oresme's mathematical intuition. In a different paper, all the present three authors discuss Oresme's understanding of affinities and the parallels between some themes in contemporary mathematical modeling and ideas present in Oresme's work [31]. Nicole Oresme was an extraordinary scholar, who anticipated many developments ahead of his time. In the present paper, we aim to add one more piece of evidence to his file.

3. DE CONFIGURATIONIBUS

Our research is based on investigating Marshall Clagett's critical edition of the treatise *De configurationibus* [20], published in 1968. We should start by briefly describing this important medieval volume. *De configurationibus* has 93 chapters, being composed of three parts. In the first part, Oresme sets the foundation for a "doctrine of configurations" and applies the doctrine to qualities, focusing on the "entities" which are permanent or enduring in time. While discussing these elements, he suggests that his theory could explain numerous physical and psychological phenomena. In the second part, Oresme describes how graphical representation can be applied to “entities that are successive”, particularly referring to motion. He concludes this part with examples illustrating the application of his theory to psychological effects, focusing on describing perceptions typically deemed as magical. Finally, in the third part, Oresme describes the external geometrical figures used to represent qualities and motions. He explains that, by comparing the areas of such figures, one may have a basis for the comparison of different qualities and motions. Here, we have briefly summarized what he called “the doctrine of configurations,” which can be viewed as an early theory of functions and of their graphical representations. Ultimately, Oresme’s doctrine was formed on solid intuition as the rigorous algebraic notation and language of functions had not been developed. Oresme’s insight into various topics in mathematics emphasizes the historical context of Oresme’s work to highlight their potential applications.

4. NICOLE ORESME AND ARISTOTLE’S WORKS

Nicole Oresme was much indebted to the major intellectual paradigm of his century, which was the recovery of Aristotle. To better place these ideas in such a context, we refer to Dirk Struik, who writes: “The study of the Aristotelian categories of quality and quantity focused attention on the difference between the *intensio* and *remissio* of a quality and the increase and decrease of a quantity,” concluding that Oresme’s pursuit “gave rise to the applications of quantitative ideas to qualities.”

Oresme was inspired by Campanus (cited by Clagett [20]) who writes: “whatever ratio is found in one hind of continuum, the same ratio can be found in all others. For just as one line is related to any other line, so any surface is related to some other surface, and any body to some other body, and similarly for time.” Additionally, Oresme pulled from Aristotle’s *Metaphysics* (more precisely with Bk. X, Chap. I, 1052b), which is interpreted by Oresme in his *Questions on the Physics* in the following way:

> ... measure, ratio, comparison, equality, inequality, etc. are initially found in quantity and [then] transmuted to all other things by means of similarity to this quantity, either extended or discrete quantity. From this it is evident by corollary that comparison is initially in quantity and

> secondarily in [those] species of quantity like angles, and thirdly in qualities with respect to their intensity... Then finally I say by way of conclusion that in every comparison it is necessary to imagine extension, intensity, discreteness, or order, and I say that intensity is always imagined by means of extension. [20]

This connection with Aristotle's work clearly illustrates how Oresme builds upon Aristotle's conclusions, and applies these ideas to a larger array of concepts where proportions could be used. Some of the concepts he is interested in are today considered part of mathematics, some part of physics or even psychology. Of particular importance is Nicole Oresme's debate with astrologers [6]. In discussing French Court's culture, C.C. Willard, while presenting Christine de Pizan's biography, points out that:

> In speaking of Charles V's interest in scientific speculation, Christine was undoubtedly recalling her father's role in his service, but she also mentions Nicole Oresme's *Book of the Sky and the World*, a translation and commentary based on Aristotle's treatise on cosmology prepared for the royal library in 1377. This text provided the most detailed and accurate analysis of terrestrial dynamics from the Greek astronomers to Copernicus. [38]

C.C. Willard adds that Oresme "dared to raise the question of the constant motion of the spheres before Copernicus," and that the "treatise was never published," so Copernicus's awareness of its existence is unknown.

As evidenced in his writings, Oresme studied various fields, from mathematics to physics to psychology. As a result of his wide coverage, Oresme's insight was potentially overlooked. In addition, during this period, Europe witnessed a multitude of revolutions [32]; hence, Oresme worked during an era of political unrest - offering another explanation for why his works were grazed over. At first glance, some of his ideas do not seem to offer mathematical insight. We hone in on particular examples that pertain to mathematical biology and modeling - fields that were developed within the past few decades.

5. ORESME'S APPLICATIONS

The generality of Oresme's doctrine resides in his efforts to model various phenomena. In *De configurationibus* we encounter his first attempt to apply his doctrine. In Chapter I.xxiv he writes:

> It is manifest from natural philosophy and experience alike that all natural bodies determine in themselves their shapes, as, for example, animals, plants, some stones, and the parts of [all of] these. They also determine in themselves certain qualities which are natural to them. In addition to their shape that these qualities possess from their subject, it is necessary that they be figured with a figuration which they possess from their intensity - to employ the previously described imagery. [20]

Thus, the method of figurations developed before is meant to serve a series of representations associated to various phenomena. Only for this remark Oresme's work would be of major interest for the historian of mathematics, as such a description anticipates several developments of modern applied mathematics. However, there is more here, because each of these figurations has intensity, so additionally their shape needs to be understood. This idea is mathematical in nature and represents concepts ahead of Oresme's times.

For our inquiry, the key point resides in Chapter I.xxxi, where Oresme writes "On difformity in cognitive powers." This is a matter that today could be associated with either psychology or physics of the brain. He writes:

> Accidents of the sensitive soul are, in accordance with the extension of the subject, figured in the organs with respect to uniformity and difformity in completely the same way as are sensibles or the other qualities [...] [20].

And he continues:

> … forms impressed in the exterior senses pass away immediately when the sensibles are withdrawn. But in the interior sense likenesses or forms

> remain even in the absence of the sensibles, just as an imprint remains in wax after the seal has been removed. However, the organ of the interior sense is not in itself differently shaped according to quantity by the impressed species or form but is only figured qualitatively in the same way that the corporeal figure of the eye is not in itself changed by receiving a species of color. [20]

What "accidents of the sensitive soul" does Oresme have in mind? His century offered him a wide array of personal experiences. The historical context of the major tragedies and experiences Oresme must have been subject to is partially an explanation of why this treatise is structured as such, and why these problems are discussed in relation with mathematical concepts. We conclude that his work is a reflection of the 14th century, particularly corresponding with the beginning of the Hundred Years War and the Black Death.

However, the French court proved to complicate the creation of Oresme's doctrine. C. W. Coopland writes:

> As much of Oresme's energy was directed to attach on "occult" practices and towards defining the bounds between what we should now distinguish as astronomy and astrology, respectively, a word may be said as to the prevalence of superstition in his day. It was clearly widespread and mischievous. We need not stress too heavily evidence drawn from the titles of the books collected by Charles V in the great library installed by him in the Louvre. [6]

In part, Oresme was motivated to form his doctrine to contradict politicians that tried to control king's mind through superstitions and astrology. However, Oresme also sought to create a consistent "doctrine, which is mathematical in nature and substance."

Oresme describes the matters of the "soul" as analogous to physical perceptions. In Oresme's words:

> … consider leather in which writing or the image of some seal is impressed. [...] Because of the nature of the leather or the quality of the

> material and the strong impression, it happens that afterwards the leather can not be stretched without the impression always remaining visible. [...] Just as it is in the case of this quantitative figuration, so similarly we ought to imagine it to be in the case of qualitative figuration of sensitive power, and, in a certain corresponding way, in the case of spiritual figuration of intellective power. [20]

Thus, the main challenge set for the researcher of the "intellective power" is to understand the causes of "visions of the soul" (Chapter I.xxxiii). In this chapter, the "things perceived by a soul through a vision, which is foreseen in dreams or in an ecstasy or excess of the mind," are compared to images reflected into "a mirror." However, mirrors could be "pure and clean" or "contaminated and infected." Other mirrors are discussed in Chapter I.xxxv, titled "On certain differences in visions," as Oresme pursues his analysis: "certain mirrors are uniform and plane or straight, and others are curved. And the curved ones are of two kinds: either concave (and these magnify the object) or convex (and in these the object appears smaller in size). [...] It is the same also in regard to souls." [20]

As shown in his analysis, Oresme needed the concepts of convexity, difformity, and curvature to investigate and explain perceptions. At stake: the ultimate truth. By developing a sound doctrine, Oresme had the potential to influence king's views and disprove dangerous political advisors. Oresme's quest started as a question of intellectual nature - the relationship of senses with the real world. This question was reduced to a psychology problem that was addressed quantitatively.

6. PLEASURE AND IMAGINATION

Chapter I.xxx is called *On the causes of the pleasure of sense and [the pleasure] of imagination*, and we can read in it that:

> … certain things are unpleasant to the human touch which would be pleasant to the touch of another animal. And similarly, within the same species, some touchable objects displease one individual but please another. The same thing is true of taste. For example, thorns [unpleasant for man]

> please the ass, while pottage [tasty to man] displeases the ass. Also, something pleases a well man and is displeasing to a sick one, and there are many such examples.

In the same chapter, we find the following idea:

> … in regard to pleasure and displeasure of human imagination or cogitation, it is manifest that certain people take delight naturally in imagining disgraceful things and in obscene talk, filth, and deformed sounds and figures. And this is a sign of the vileness of their constitution with respect to the ratio and figuration of their natural qualities. On the other hand, in contrary fashion, others abhor foulness and naturally like purity, decency and honesty. And this is a sign of the good configuration of their natural qualities and of their noble constitution.

What is unusual in this fragment is that Oresme speaks of "ratios and figurations" associated to the "natural qualities" of a human being. Although at the middle of the 14th century the scholar does not have any statistical data available or present numerical findings in his work, Oresme discusses mathematical examples of pleasure.

Two recent studies conducted by O'Doherty *et al*. [19] and McClure *et al*. [16] have started to shed light on the principles of pleasure prediction from a neuroscience perspective. In modern times, a thorough scientific discussion on pleasure and imagination is directly related to the understanding of brain's structure and neural activity. Braver and Brown [3] write that: "Over the last decade, however, there has been a rapid acceleration of scientific progress in this area [pleasure prediction], based in large part from two parallel streams of research: neurophysiological studies of conditioning in nonhuman primates and noninvasive neuroimaging studies of reward processing in healthy humans." Modern neuroimaging begins to align with Oresme's explanation of pleasure and human behavior. As Braver and Brown explain: "the work discussed here [16, 19] might be fruitfully applied toward an understanding of the neural basis of normal individual differences (*e.g.*, personality traits) and pathologies of reward learning (*e.g.*, gambling and addiction), as these areas of study are still in their infancy."

Other investigations in recent, and somewhat unexplored areas of research have started to shed some light on "the causes of the pleasure of sense and [the pleasure] of imagination." These investigations include studies of desire, motivational salience, and incentive salience. Neuroscience and neuropsychology are beginning to understand the workings of dopamine, which is the primary neurotransmitter in charge of mediating pleasure, motivation, and reward. In a recent study, Zhang *et al*. [41] start their inquiry from the assertion that "incentive salience is a motivational property with `magnet-like' qualities." Incentive salience is a tricky concept, but it is roughly defined as the process of feeling pleasure from obtaining a rewarding stimulus using an adequate motivator. An example is pottage. For man, pottage is an adequate motivator to feeling the pleasures of dinning a tasty meal. For an ass, the contrary is true, which shows the importance of choosing the "correct" motivator to obtain an adequate pleasurable response. This resonates with Oresme's statements on how "some touchable objects displeases one individual but pleases another." Zhang *et al*. write: "When attributed to reward-predicting stimuli (cues), incentive salience triggers a pulse of `wanting' and an individual is pulled toward the cues and reward. A key computational question is how incentive salience is generated during a cue re-encounter, which combines both learning and the state of limbic brain mechanisms."

This question calls for approaches from the field of computational mathematics. Some of the themes presented by Oresme in *De configurationibus* correspond to modern approaches involving computational viewpoints. The concept of incentive salience, found in Zhang *et al*.'s study, is "essentially a Pavlovian motivational response: it takes Pavlovian associations as its learned input." The authors "aim to account for such motivation variations by identifying incentive salience computations that dynamically determine Pavlovian motivation value. These computations must integrate Pavlovian learned inputs with the status of the brain mesolimbic mechanisms that reflect current physiological states (hunger, drug intoxication, sensitization, etc.)."

Over the past few decades, there have been attempts to model pleasure and imagination, particularly to understand learning processes or brain's structure. Russell [28] presents one of the founding papers on this topic, which puts forward one of the first mathematical models of "affect" (the set of psychological attributes of pleasure, displeasure, stress, depression, excitement, etc.). The works by Cabanac *et al*. [5] study the role of pleasure in decision making, while Tsuda *et al*. [34] investigate the role of the hippocampus in episodic memory and imagination. Addyman and French [2] thoroughly discuss computational modeling of cognition, while Ovsich [22] describes "mathematical models of desire, need and attention." In [35], Thilakarathne models intention, attention, and awareness while, in [36], the same author focuses on a parameter estimation method for dynamic computational cognitive models, a study continued in [37].

In 2017, Mattek *et al*. write [17]: "Although it is possible to observe when another person is having an emotional moment, we also derive information about the affective states of others from what they tell us they are feeling. In an effort to distill the complexity of affective experience, psychologists routinely focus on a simplified subset of subjective rating scales (*i.e*., dimensions) that capture considerable variability in reported affect: reported valence (*i.e*., how good or bad?) and reported arousal (*e.g*., how strong is the emotion you are feeling?)". The complexity and importance of developing performant models capturing the "subject affect" are difficult to overstate. Oresme worked with such "qualities" as natural candidates for quantification, and only recently have such themes been highlighted in contemporary scientific literature.

7. FEAR, HOPE, SORROW AND JOY

In Chapter II.xxxvii, Oresme writes *On the causes of certain effects arising in the subject itself, based on the prior statements*, where he approaches the theme of subjective perceptions and associates them to quantities. This idea was bold for his time and a challenge today for our contemporary scholars working in applied mathematics. The fragment we would like to refer to is the following:

> Apprehension, cogitation, or imagination changes the body of the person who is apprehending, and particularly by reason of accompanying desire or passion. For if someone thinks strongly about revenge, and does so

> with intense excitement, his blood is stirred and his face is changed. It is the same with respect to fear, joy, and other accidents of the soul. Also, because of the variety of these accidents people produce different effects externally.

The fact that Oresme associates quantifications to these subjective perceptions is expressed in a fragment as the following, in the same chapter,

> … it is possible that movement or passion in the body is varied not only because of greater or lesser intensity of imagination or affection but also because of a diversity as to difformity in the figuration of the aforesaid accidents of the soul. For example, if someone imagines [something] with affection or thinks about revenge, and the difformity of this cogitation or imagination is duly figured, then the act will duly carry out the commands and he will be as one particularly fortunate in carrying out or executing his intention. But if the imagination or affection is not duly figured, then the act will not be duly performed even though the imagination or affection is sufficiently intense. And in the same way one ought to speak of a species of love or hate and similarly of other accidents of the soul. For in this matter one ought to speak of motions of the soul in the same way as we spoke of motions of the body in chapter ten of this [part] and of the beauty of the figuration of velocities in chapter eleven.

The idea is clear: the "motions of the soul" should be viewed in the same way we view the "motions of the body." In modern language, we should assess emotion dynamics using approaches from mechanics or fundamental physics. The fact that quantification is possible is understood by using these examples in an overall "doctrine of configurations," which is nothing short of an early theory of functions. The very idea of quantifying variations in the realm of emotions was used extensively by Oresme in *De configurationibus* among his examples.

We are now interested in assessing how current research approaches the idea of quantifying such "accidents of the soul," (*e.g.* fear, hope, sorrow, or joy) and demonstrate how current investigations align with Oresme's curiosity.

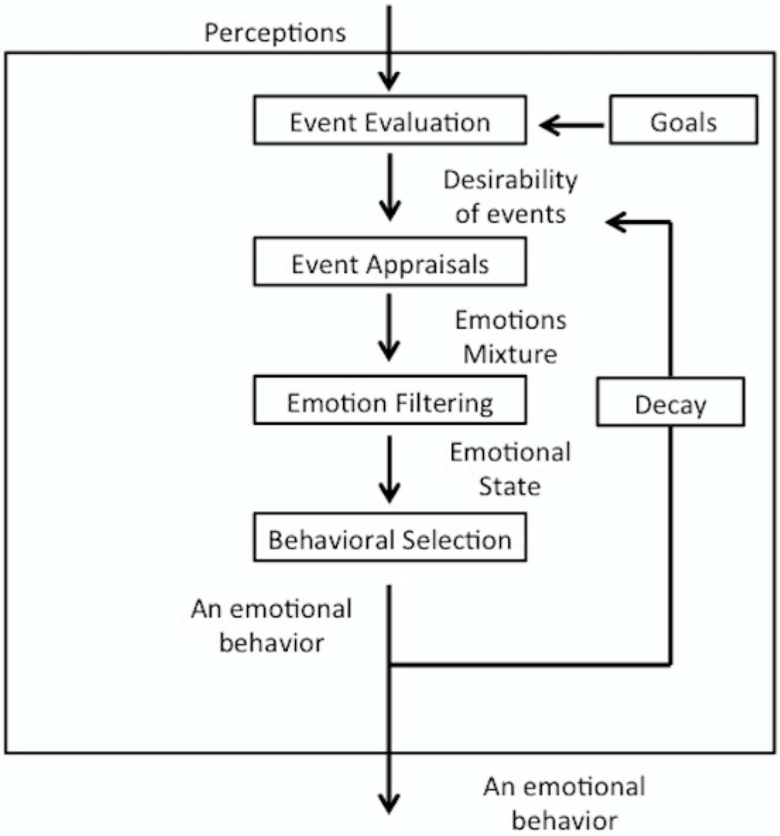

Fig. 1. The Emotional Component in Decision Making.

This figure illustrates the process by which perceptions are evaluated and, with reference to a current emotional state, a behavioral decision is made.

In a recent paper [4], Broekens, Jacobs and Jonker focus their efforts on “joy, distress, hope and fear” in the context of “the relation between adaptive behavior and emotion.” The authors describe their approach as follows: “Using the reinforcement learning framework, we propose that learned state utility, $V(s)$, models fear (negative) and hope (positive) based on the fact that both signals are about anticipation of loss or gain.” The computational model proposed by the authors shows how hope, fear, joy, and distress can be appropriately quantified and used to replicate behavioral dynamics of human emotions. This is an example of interesting work in the areas of emotion dynamics and affective computing - an emerging field.

In a comprehensive inquiry [29] published in 2000, El-Nasr, Yen, and Ioerger discuss the importance of emotions in the decision-making process. They write:

“Emotions are an important aspect of human intelligence and have been shown to play a significant role in the human decision-making process. Researchers in areas such as cognitive science, philosophy, and artificial intelligence have proposed a variety of models of emotions. Most of the previous models focus on an agent’s reactive behavior, for which they often generate emotions according to static rules or pre-determined domain knowledge. However, throughout the history of research on emotions, memory and experience have been emphasized to have a major influence on the emotional process.” Although we seem to enter a territory where mathematics has little to say, El-Nasr, Yen, and Ioerger “propose a new computational model of emotions that can be incorporated into intelligent agents and other complex, interactive programs. The model uses a fuzzy-logic representation to map events and observations to emotional states. The model also includes several inductive learning algorithms for learning patterns of events, associations among objects, and expectations.” This fuzzy logic adaptive model of emotions (FLAME) also included computer user software interface PETEEI (Pet with Evolving Emotional Intelligence), which emulates model’s outcome using a variety of emotional inputs (*e.g.* desire, joy, and anger) and a numerical integration routine of previous and current emotional states, as shown in Figure 1. Other recent efforts in the fields of applied mathematics and computer science include SHAME [24], EMA [13], SOAR [18], EIA [1], and GEMA [14], all of which were built from the OCC [21] categorization of emotions. All these papers investigate the role of emotions in human intelligence. In [14], one fundamental reference is Marvin Minsky’s classical work *The Society of Mind*, which shows that the question of investigating the dynamics of emotions an age-old question considered in various cultures and contexts. Presently we see a multitude of mathematical models working with these ideas, and beginning to align with Oresme’s original intuition.

Along the same lines as above, the work by Rodrigues, Ramos, and Wang [26] presents “a comprehensive survey of cognitive and computational models of emotions” which they explain is the result of multidisciplinary studies across several scientific disciplines. This is important because it describes clearly that such a topic ranges presently in a multidisciplinary territory. The authors explain how “emotions are currently the focus of study in multiple disciplines such as psychology, neuroscience, philosophy, computer science, cognitive sciences, and cognitive informatics [...]. This multidisciplinary inquiry has provided evidence that shows the significance of emotions not only to the rational behavior of individuals, but to achieve more believable and human-like behaviors in intelligent systems." In a paper published in 2016, in the field of artificial intelligence (AI), Kowalczuk and Czubenko [15] describe their interest in AI as follows: "One of the major differences between a human and a humanoid robot relies on the feeling and expressing of emotions. For this reason, robots appear to be heartless to people. To reduce this appearance, a considerable number of robot-oriented projects have begun to take into account the handling of emotions, especially to simply simulate them (without whatever internal effects). As far as emotions are concerned, there are several projects which take into account the issue of expressing emotions by a robot." Consonant to this vision, Perez-Gaspar, Caballero-Morales, and Trujillo-Romero [23] write that: "Service robotics is an important field of research for the development of assistive technologies. Particularly, humanoid robots will play an increasing and important role in our society. More natural assistive interaction with humanoid robots can be achieved if the emotional aspect is considered. However emotion recognition is one of the most challenging topics in pattern recognition and improved intelligent techniques have to be developed to accomplish this goal." It is evident from their conclusions that the subject of "emotion recognition" will be of interest in the following years, and the related problem of quantifying various values related to it will be in scholars' attention for the years to come.

The works most reflective of Oresme's ideas can be found in [9, 10, 40], where the focus is on *measuring happiness*. In these studies, the authors point that the works on quantifying happiness is recent, and they cite a large array of specialized works addressing this quest. In [9] they write: "Since the early 1990s, researchers in the movement of positive psychology have encouraged the study of the conditions and processes which contribute to the optimal functioning of individuals, groups, and institutions." In [10], the authors used a continuous scale to quantify happiness levels and in [40] the author used a continuous differential equation model to understand the relationship between happiness and health. The latter works illustrate recent efforts to quantify Oresme's "accidents of the soul" and how based on "the variety of these accidents people produce different effects externally." However, these external effects are difficult to measure, as Rutledge *et al*. [27] explain: "Although social comparison is a known determinant of overall life satisfaction, it is not clear how it affects moment-to-moment variation in subjective emotional state... [an individual's] subjective emotional state reflects not only the impact of rewards they themselves receive, but also the rewards received by a social partner." Thus, we survey recent investigations, however, even for these topics we can find analogies in Oresme's work.

8. CONCLUSIONS

The examples presented in the previous sections illustrate how visionary Oresme's work was. By using examples from the domain of subjective perceptions, while using his doctrine of configurations, Oresme simultaneously attracted the attention of his readers and addressed themes of interest today. This shows that there are many similarities between a series of developments in contemporary mathematical modeling and the examples used by Nicole Oresme in *De configurationibus*. As far as we are aware, no other parallels have been described in the scientific literature, and none of the works in applied mathematics we cite indicate Nicole Oresme among their references. No direct affiliations can be claimed between Oresme and developments in applied mathematics. What is certain is that Oresme's works are deeply rooted in Aristotle's natural philosophy, and that this Norman-French medieval author endowed with an unusual intuition approached and raised a series of questions that are of interest today within the realm of applied mathematics.

Authors' contributions: All three authors had equal contributions to this study.

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