Memoirs of the Scientific Sections of the Romanian Academy
Tome XXXIX, 2016

BOOK REVIEWS

A.S. FOKAS, B. PELLONI (Editors), *Unified Transform for Boundary Value Problems*, Applications and Advances, Other Titles in Applied Mathematics 141, SIAM, Philadelphia, 2015, XI + 282 p., ISBN 978-1-61197-381-5.

This book describes the state-of-the-art in the developments and applications of the uniform transform method to the analysis and numerical modelling of boundary value problems for linear and integrable nonlinear PDEs. A significant part is devoted to modern applications of the boundary element method, a well-established numerical approach for solving linear elliptic PDEs.

The book, divided into three parts, consists in eight chapters closely related among them. After an introductory chapter conceived by the editors, the first part deals with theoretical results and presents recent advances in the application of the unified transform. Explicit solution representations for several classes of boundary value problems are constructed and rigorously analyzed. Its first two chapters are dedicated to evolution problems. In this framework, in what concerns linear PDEs, a first goal is to illustrate how the unified transform method can be implemented for problems involving non-separable boundary conditions. The second one is to construct classical spectral representations in an easier way than the standard approaches. Chapter 3 deals with the relation between the unified transform approach and the classical ones to the solution of initial-boundary non-linear problems.

The remaining of the first part deals with elliptic problems, both linear and non-linear. In Chapter 4, it is shown that analysis of the Dirichlet-to-Neumann map provides a new approach to obtaining interesting results for linear elliptic PDEs. The next chapter gives a new classification of boundary conditions for the elliptic sine-Gordon equation in a semi-infinite strip that satisfies a necessary condition for realizing that a boundary value problem can be effectively linearized.

The second part presents an overview of variational formulations of second-order linear elliptic PDEs based on Green's identities, namely the weak form of the PDE, the discontinuous Galerkin methods and Trefftz-discontinuous Galerkin methods, the formulation based on a quadratic functional, the boundary integral equations and the null-field method.

Part III treats numerical applications, mentioning that the unified transform has inspired numerical techniques for determining the unknown boundary values. For elliptic PDEs in the interior of a polygon, this technique provides the analogue of the boundary integral method, but with the spectral space instead of the physical space. The boundary value problem outside of a bounded domain is also discussed, with applications of the boundary element method to high-frequency problems occurring in acoustics. A unified numerical approach for nonlinear Schrödinger equations is introduced in the last chapter, where it is shown that there exists an efficient numerical technique for the solutions of matrix Riemann-Hilbert problems.

This book is suitable for scientists working on boundary value problems in physics and engineering, but also for applied and numerical researchers in PDEs.

Adrian Zălinescu

JOSEF MÁLEK, ZDENĚK STRAKOŠ, *Preconditioning and the Conjugate Gradient Method in the Context of Solving PDEs*, SIAM Spotlights 1, SIAM, Philadelphia, 2015, X + 104 p., ISBN 978-1-611973-83-9.

This book provides a roadmap from an applied problem to its numerical solution, passing through modelling, analysis, discretization and solution of the discretized problem. The main concept employed in this text is preconditioning of the conjugate gradient method, which is in this case developed in connection with PDE analysis, unlike the traditional method, based on the preconditioned finite-dimensional algebraic system. In order to carry out the task, the authors appeal to techniques of functional analysis, calculus of variations and matrix iterative computation using Krylov subspace methods. The text brings also a constructive critique to some commonly accepted views, by addressing current misconceptions and making provoking open questions for further research.

The first introductory chapters accustom the reader not working in the field of PDEs to some of its concepts. The classical and weak formulation of linear elliptic PDEs is analyzed in a first step. Standard tools of functional analysis, such as Riesz representation theorem and Lax-Milgram lemma, are then discussed.

In Chapter 4, the concept of preconditioning is introduced, by defining it from the choice of an appropriate inner product with the associated Riesz map, which transforms the functional equation under study into a more suitable form for an efficient numerical solution. The next chapter presents the conjugate gradient method in Hilbert spaces, providing also the link among the Vorobyev moment problem, conjugate gradient and the Gauss-Christoffel quadrature. The sixth chapter analyses how the conjugate gradient method can be fully described in Hilbert finite-dimensional spaces in terms of vectors and matrices.

After providing the notation for Galerkin discretization in Chapter 7, the subsequent one is devoted to the description of preconditioning of matrix systems as an interpretation for transformation of the discretization basis. Chapter 9 recalls consistency, stability and convergence of discretization schemes. Evaluation of errors in numerical PDE computation is dealt with in Chapter 10. The last chapters are concerned with critical discussions on the limitations of condition number-based descriptions, as well as usual errors due to inexact algebraic computations.

The book will certainly appeal the researchers interested in the above issues, being intended not only for mathematicians, but for engineers, physicists and chemists, as well.

Adrian Zălinescu

DMITRI KUZMIN, JARI HÄMÄLÄINEN, *Finite Element Methods for Computational Fluid Dynamics, A Practical Guide*, Computational Science and Engineering 14, SIAM, Philadelphia, 2015, VIII + 313 p., ISBN 978-1-61197-360-0.

This book introduces the reader into one of the most powerful numerical techniques, the finite element method, in the context of computational fluid dynamics (CFD), the art of solving partial differential equations modelling the motion of fluids, as well as mass and heat transfer phenomena.

The book is accessible to a large audience, who will find a clear and detailed presentation of the state-of-the-art numerical algorithms for convection-dominated transport problems and the incompressible Navier-Stokes equations. Stress is therefore laid on practical implementation rather than on burdening the reader with underlying mathematical theory. The reader is guided through all stages involved in solving the equations of fluid mechanics using continuous finite elements, the complexity of the text growing in small steps, from a basic level to difficult flow problems.

The text is structured into nine chapters, the first ones being written for beginners without knowledge on CFD or finite element method. Chapter 1 is devoted to basic notations and mathematical foundations. The second one deals with the general CFD design philosophy using a scalar transport equation as a toy problem, and prepares the reader for numerical methods for partial differential equations. He/she is already encouraged to perform numerical experiments using the finite element library Elmer, presented at the end of this chapter. In the third chapter, Navier-Stokes equations are derived from physical conservation principles, using moving finite and fixed infinitesimal control volume models. An introduction to Galerkin finite element method is given in Chapter 4. The presented concepts are then applied to the one-dimensional heat equation. The construction of basic finite element approximations to the two-dimensional heat equation and to the steady incompressible Navier-Stokes equations for laminar flows is addressed in the next chapter. In

Chapter 6, the numerical treatment of convective terms in the context of one-dimensional model problems discretized using linear finite elements on uniform meshes is discussed while, in the following chapter, multidimensional extensions and generalizations are presented. Chapter 8 focuses on the design of stable finite element approximations, fractional-step algorithms and efficient iterative methods for the time-dependent incompressible Navier-Stokes equations. Coupling with the transport equations of the k-ε turbulence model and the implementation of wall laws in finite element codes are also addressed.

In Chapter 9, in addition to numerous citations throughout the book, the interested reader will find a list for further reading.

This guidebook is particularly well-suited for students in computational engineering, while physicists, computational scientists and developers of numerical simulation software will equally find in this guidebook a valuable source of information.

Adrian Zălinescu

GREGORY ESKIN, LEONID FRIEDLANDER, JOHN GARNETT (Editors), *Spectral Theory and Partial Differential Equations*, Conference in honor of James Ralston's 70th birthday, University of California, Los Angeles, CA, USA, June 17–21, 2013. Proceedings. Contemporary Mathematics 640, AMS, Providence, 2015, IX + 197 p., ISBN 978-1-4704-0989-0.

This volume, dedicated to prof. James Ralston on the occasion of his 70th birthday, contains the Proceedings of the Conference on Spectral Theory and Partial Differential Equations, held at the University of California, Los Angeles between June 17–21, 2013. The nine papers in this volume are discussing topics of interest in spectral theory and partial differential equations, such as: inverse problems, minimal partitions and Pleijel's Theorem, spectral theory for a model in Quantum Field Theory, beams on Zoll manifolds.

The subject of the first article is the application of ideas from noncommutative geometry to inverse problems on manifolds, namely recovering the manifold *via* relevant measurements at the boundary area. In order to achieve this, the author uses the boundary control method for determining an appropriate algebra from inverse data.

The second paper is concerned with the study of mathematical models based on the Quantum Field Theory, without restrictions for the spins of the particles involved. The author applies Weinberg's formalism to construct a mathematical model based on the weak decay of hadrons and nuclei. With the application of the commutator theory, he obtains a limiting absorption principle for evidencing that the spectrum of the associated Hamiltonian operator is absolutely continuous above the energy of the ground state and below the first threshold.

In paper 3, the authors review the properties of minimal spectral k-partitions in the two-dimensional space. They also explore the link between the proof of Pleijel's Theorem and the lower bounds for the energy of a partition. Some consequences of the magnetic characterization of minimal partitions for critical points are also discussed.

The fourth article deals with stability estimates for the near field of a radiating solution of Helmholtz equation from the far field. These estimates present increasing stability of recovery of the near field from the scattering amplitude with growing frequency.

The next paper is devoted to the study of the inverse scattering associated with n-dimensional asymptotically hyperbolic orbifolds having a finite numbers of cusps and regular ends. The authors show that the generalized S-matrix, which is introduced by observing the Fourier coefficients of solutions of Helmholtz equation at a cusp, determines the manifolds and orbifold structure.

The object of the sixth article is the asymptotic approximation of the semi-classical Schrödinger equation in periodic media using Gaussian beams. A superposition of the Bloch-band based on such beams for generating high frequency approximate solutions to the original wave field is formulated.

The following article deals with randomly weighed Sobolev inequalities on Euclidian spaces and their applications, starting from the idea that introducing some randomness improves the deterministic Sobolev type inequalities. For the sake of simplicity, the properties of the harmonic oscillator and Hermite functions are analyzed.

In the eighth paper, the authors focus on Maxwell's equations in a waveguide, with a particular interest on global uniqueness in determining the conductivity, permeability and permittivity by a partial Dirichlet-to-Neumann map limited to an arbitrary sub-boundary.

Gaussian beams on Zoll manifolds and maximally degenerate Laplacians are the subject of the last article in the volume. Here, one of the goals is to find obstructions to constructing Gaussian beams solving the eigenvalue equation to arbitrarily high orders, and to analyze how they can vanish in the maximally degenerate case. The second one is to show why these constructions are possible for Zoll Laplacians even in the case they fail to satisfy the non-degeneracy assumptions.

This volume is recommended to researchers and graduate students with interests in spectral theory, inverse problems, mathematical physics, functional analysis and PDEs.

Adrian Zălinescu

JOHN ROE, *Winding Around: The Winding Number in Topology, Geometry and Analysis*. Student Mathematical Library 76, AMS, Providence, 2015, XIII + 269 p., ISBN 978-1-4704-2198-4.

This book provides a clear exposition on the broad applications a simple intuitive geometric notion, the winding number, has in various fields of mathematics. Although very clearly expressed in common language as the number of times that a closed curve circumvents a given point, to define it in an accessible, yet mathematically sound way, is not straightforward. However, the author accomplishes this mission in a remarkably pleasant manner and then carries the reader on a voyage through topology, differential geometry, functional analysis and algebraic topology, punctuated with important results such as the fundamental theorem of algebra, the Jordan curve theorem, the Hopf index theorem, the Toeplitz index theorem and the Bott periodicity theorem.

The text is organized into nine chapters and no less than seven appendices. The first chapter serves as an introduction, while the second presents the preliminary notions and results in homotopy, needed to define the winding number. This task is carried out in the third chapter, by straightening out the smooth loop near each intersection point with straight lines. One of the first applications given here is that a polynomial of order n has exactly n complex roots (the fundamental theorem of algebra).

The fourth chapter presents some classical theorems in the topology of figures in the plane, culminating with the answer to the question why every simple closed curve has an inside and an outside part (the Jordan curve theorem). The fifth chapter focuses on the link between the winding number and the integration in the complex plane, providing a hint to introducing higher-dimensional topological structures. The next chapter deals with rotation numbers of smooth, regular paths and vector fields on surfaces. By using these notions, the author is able to give the statement and the proof of Hopf index theorem, which asserts that the sum of indices of the singularities of a smooth tangent vector field on a compact oriented surface is equal to the Euler characteristic of the surface.

Chapter 7 extracts the reader from the finite-dimensional setting, by relating the winding number and analysis of Fredholm operators on Hilbert spaces with Toeplitz index theorem. In the eighth chapter, understanding and classifying loops in the punctured complex plane with the aid of the winding number are generalized to arbitrary metric spaces by introducing the fundamental group of homotopy classes. This notion is further extended to homotopy groups in the last chapter, crowned by Bott's periodicity theorem, which contradicts the intuitive idea that the homotopy groups of reasonable spaces should become successively more and more difficult to compute.

The author's style of writing is succinct, but clear and expressive. The final-year undergraduate courses are sufficient for tackling the subjects covered by the book. This makes it accessible to students interested in the notion of the winding number and its occurrence in areas of mathematics such as topology, analysis and differential geometry.

Adrian Zălinescu

FIRAS RASSOUL-AGHA, TIMO SEPPÄLÄINEN, *A Course on Large Deviations with an Introduction to Gibbs Measures*, Graduate Studies in Mathematics 162, AMS, Providence, 2015, XIV + 318 p., ISBN: 978-0-8218-7578-0.

This book constitutes an introduction to the theory of large deviations, which is the theory of computing asymptotics of probabilities of rare events. It also combines large deviations with equilibrium statistical mechanics *via* Gibbs measures.

The book consists of three parts, sub-divided, in their turn, into 16 chapters, and three appendixes. The first part covers the general large deviation theory (Chapters 1–3), elements of convex analysis (Chapter 4), and the large deviations of independent and identically distributed processes (Chapters 5 and 6). Analysis of the last aspect follows three levels: Cramér's theorem, Sanov's theorem, and the process level large deviation principle for independent and identically distributed variables indexed by a multidimensional square lattice.

Gibbs measures are introduced in the second part (Chapter 7); translation-invariant Gibbs measures are characterized by the Dobrushin-Landford-Ruelle variational principle (Chapter 8). In Chapter 9, the phase transition of the Ising model is considered; Chapter 10 deals with the Fortuin-Kasteleyn random cluster model and the percolation approach to Ising phase transition.

In the last part, the large deviation themes of Part I expand into several directions. Moderate deviations and more precise large deviation asymptotics are now given for independent and identically distributed random variables (Chapter 11). The Gartner-Ellis theorem is attentively developed in Chapter 12. The next step consists in moving from independent and identically distributed processes to Markov chains, to non-stationary independent random variables, and to random walk in a dynamical random environment by the theorem of Baxter and Jain (Chapters 13 and 15–16).

Reading of the textbook is based on frequent exercises; the authors have also included a guide to dependencies between different parts of the book. The appendixes give a quick overview of some basic results of analysis and probability, but also consist of specialized results used in the text, such as a minimax theorem and inequalities from statistical mechanics.

This book is intended for an one-semester course, suitable to graduate students interested in probability, the theory of large deviations, and statistical mechanics, but it will certainly appeal more experimented researchers in these areas.

Adrian Zălinescu

A.J. ROBERTS, *Model Emergent Dynamics in Complex Systems*, Mathematical Modelling and Computation 20, SIAM, Philadelphia, 2015, XII + 748 p., ISBN 978-1-611973-55-6.

This book examines emergent dynamics in complex, nonlinear dynamical systems in the context of invariant manifolds and coordinate transformations, with special attention to normal form theory, and the theory of centre and slow manifolds. Stress is laid on the geometric interpretation of the modelling process, while the corresponding geometry allows determination of strengths and weaknesses in the construction and analysis of a model.

The book is divided into seven parts, organized in 21 chapters; each part is followed by a short summary, which outlines the key concepts and results analyzed. The first part introduces the perturbative techniques designed for the derivation of asymptotic series solutions to algebraic and differential equations. The approach followed by the author is iterative, and is driven by the so-called residual of the governing equations. Nonlinear coordinate transformations are also introduced, and then related to the normal form theory.

The essential concepts of centre manifold theory are developed in the second part, with emphasis on its importance for the accurate reduction of real-world dynamical systems. Slow manifolds are defined as centre manifolds which arise due to zero eigenvalues in the corresponding linearization. Three principles are present here: the existence of an appropriate model; its relevance and emergence as an (exponential) attractor; its construction to any specified error (for which a general, coordinate-free algorithm is presented).

The third part is concerned with modelling on separate spatial scales, through which the large-scale dynamics captures micro-scale effects. The author introduces the fundamental partial differential equations of fluid dynamics, together with examples: the flow of a viscous fluid along a pipe and of a thin film of fluid over a solid substrate. The centre manifold theory is applied for the identification of lateral space derivatives as natural small parameters, Fourier analysis providing a rigorous justification of the approach.

Part four discusses the role in mathematical modelling of coordinate transformations, as well as of the resulting normal forms; the subjectivity of the definition of a normal form is emphasized. The existence and emergence of centre manifolds is immediately established with the aid of a nonlinear analogue of matrix diagonalization. Finally, three examples for the controversial concept of “sub-centre slow manifolds” are provided; general criteria of existence of such manifolds are then given. The centre manifold techniques developed in this part are adapted in the following one, in order to construct efficient schemes for the numerical solution of PDEs. In this way, an alternative approach for the derivation of accurate (spatial) discretization in which the governing equation itself resolves the subgrid scale field yield, in contrast to conventional techniques is proposed. Centre manifold theory is seen to guarantee the consistency of the emergent discretization for certain classes of PDEs.

The sixth part deals with oscillatory dynamics on centre manifolds which arises in the corresponding linearization through eigenvalues with non-vanishing imaginary parts. Technical difficulties due to the form of the homological equation - determining higher-order corrections to the centre manifold - are overcome by appropriate parametrization. It is finally shown that the onset of instabilities in delay-differential equations can also be modelled within that framework.

In the last part, noise is incorporated into the modelling process, interpreting it as random fluctuations in time. The author focuses on applications from science and engineering, by considering the method of averaging for deterministic non-autonomous systems. A basic introduction to stochastic calculus is given, the stochastic slow manifolds being uncovered by the use of time-dependent coordinate transformations. Remarkably, the average stochastic manifold is proven not equal to the deterministic one.

The large amount of material contained in the book is a valuable reference for students, engineers, mathematical researchers and practical modellers in physical sciences.

Adrian Zălinescu