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WAVELENGTH-DEPENDENT FUZZY MODELS FOR IR ATTENUATION IN FOG

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We propose a fuzzy logic-based wavelength-dependent model of IR attenuation in fog to unify the double-exponential family of models proposed by Nadeem *et al*. into a single model that can be uniformly applied to determine attenuation for various wavelengths and visibility conditions. The fuzzy logic model is compared with a model based on a polynomial interpolator.

Key words: Atmospheric attenuation, infra-red, visibility, fuzzy system interpolator, polynomial interpolator.

1. INTRODUCTION

IR wireless communication systems are strongly affected by atmosphere humidity, fog, haze etc. (Henninger, 2010), (Carruthers, 2002), (Muhammad, 2010) and (Awan, 2010). The fog, which is made up of fine water particles suspended in the air, with a diameter of less than $100\,\mu\text{m}$, is one of the most important atmospheric factors affecting the transmission of electromagnetic radiation, especially in the visible and IR spectrum. The prediction of fog attenuation is useful for FSO (Free Space Optics) links performance analysis and for adapting FSO to atmospheric conditions. In the range of IR wavelengths, electromagnetic radiation propagation through the atmosphere is affected by absorption and scattering by air molecules and aerosols (Henninger, 2010) and (Carruthers, 2002). As a result of these phenomena, in case of fog, there is a significant decrease of the signal due to effects of the beam spreading and wavefront distortion (Henninger, 2010), (Nadeem, 2010c) and (Carruthers, 2002).

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Further in this section, we briefly summarize the fundamentals of the topic, by paraphrasing the cited literature. The IR transmission in fog can be optimized by choosing an adequate wavelength, such that the attenuation is minimized. Because measurements of the attenuation in the visible range are more frequent than measurements in IR, and because visibility measurements are available from various sources, as airports and meteorological stations, it is convenient to have a relationship between IR and visible light attenuation. Such a relationship would help determine IR attenuation whenever light attenuation is known.

The IR and visible atmospheric attenuation is described by the Beers-Lambert's law (Naboulsi, 2004):

$$\tau(L) = \frac{I(L)}{I(0)} = \exp(-\gamma L) \tag{1}$$

where: $\tau(L)$ is the total transmittance of the atmosphere; $I(0)$ and $I(L)$ are the optical intensities of the source, respectively at distance L; γ is the total attenuation coefficient or extinction coefficient per unit length. The attenuation (extinction) coefficient depends on the absorption $\alpha(\lambda)$ and scattering $\beta(\lambda)$ coefficients (Naboulsi, 2004):

$$\gamma(\lambda) = \alpha(\lambda) + \beta(\lambda). \tag{2}$$

Because the IR wavelengths range is located in an atmospheric transmission window where molecular absorption is negligible and the imaginary part of the refractive index due to aerosols is very small, the attenuation coefficient is approximated by the scattering coefficient (Naboulsi, 2004).

Optical attenuation is expressed in terms of atmospheric visibility V, defined as the distance at which the transmittance falls to a given value ε (*i.e.* $\tau(V) = \varepsilon$), or the distance from which a dark object near the horizon can be distinguished. In 1924, on the basis of the intuitive threshold contrast, Koschmieder proposes $\varepsilon = 0.02$. Later, the World Meteorological Organization adopted the value $\varepsilon = 0.05$ (Prokes, 2009). The Koschmieder law of visibility is:

$$V = \frac{3.912}{\gamma_{550\text{nm}}}, \tag{3}$$

where 550 nm is the wavelength that corresponds to the maximum intensity of the solar spectrum.

The attenuation coefficient can be approximated by the empirical law (Kim, 2001):

$$\gamma(\lambda) = \beta(\lambda) = \frac{3.912}{V}\left(\frac{\lambda}{550}\right)^{-q}, \tag{4}$$

where the coefficient q depends on the particle size distribution and can be determined experimentally.

In the literature, several models describe the relationship between visibility and optical attenuation due to fog. Coefficient q is given by one model proposed by Kruse (Kruse, 1962) and another model proposed by Kim (Kim, 2001). The attenuation in dB/km for both Kruse and Kim model is given by (Kruse, 1962) and (Kim, 2001):

$$A_{Kruse\&Kim} = \frac{13}{V}\left(\frac{\lambda}{550}\right)^{-q}. \tag{5}$$

In Kruse's model q is defined as (Kruse, 1962):

$$q = \begin{cases} 1.6 & if\ \mathrm{V} > 50\,\mathrm{km} \\ 1.3 & if\ 6\,\mathrm{km} < \mathrm{V} < 50\,\mathrm{km} \\ 0.585V^{1/3} & if\ \mathrm{V} < 6\,\mathrm{km} \end{cases} \tag{6}$$

and in Kim's model as (Kim, 2001):

$$q = \begin{cases} 1.6 & if\ \mathrm{V} > 50\,\mathrm{km} \\ 1.3 & if\ 6\,\mathrm{km} < \mathrm{V} < 50\,\mathrm{km} \\ 0.16V + 0.34 & if\ 1\,\mathrm{km} < \mathrm{V} < 6\,\mathrm{km} \\ \mathrm{V} - 0.5 & if\ 0.5\,\mathrm{km} < \mathrm{V} < 1\,\mathrm{km} \\ 0 & if\ \mathrm{V} < 0.5\,\mathrm{km} \end{cases}. \tag{7}$$

Nadeem *et al.* proposed in (Nadeem, 2010a) and (Nadeem, 2010b) a generalized model for attenuation depending on wavelength,

$$A_{Nadeem} = ae^{bV} + ce^{dV}. \tag{8}$$

The expressions for the constants *a, b, c* and *d* are given by (Nadeem, 2010a):

$$\begin{aligned} a(\lambda) &= 185.8 - 0.0522\lambda \\ b(\lambda) &= -2.239 \cdot 10^{-6}\lambda - 0.002148 \\ c(\lambda) &= 25.42 + 0.01869\lambda \\ d(\lambda) &= 0.0008465 - 7.41 \cdot 10^{-7}\lambda \cdot \end{aligned} \tag{9}$$

In this article, we propose two unified models for computing the direct path IR attenuation at various wavelengths as a function of visibility. The first model is a polynomial interpolator. The second one is a fuzzy logic system that allows trimming the model when extra data is available.

The purpose of the paper is to determine a function $A(V,\lambda)$, where A represents the attenuation measured in [dB/km], V is the atmospheric visibility, and λ is the wavelength of the IR beam, such that the function is an exact interpolator for the λ-independent models proposed by Nadeem *et al*. By model interpolator we mean that for the set of λ_i values used in the models of Nadeem *et al*., the bi-variable function $A(V,\lambda)$ produces the corresponding $A(V,\lambda_i)$ models reported in (Nadeem, 2010a) and (Nadeem, 2010b). A bivariate approximation of IR attenuation on fog is useful in designing multi-spectral communication lines, for predicting the reliability of new IR communication designs, and for choosing the communication wavelength for new designs before experimental data are available.

The λ-independent models reported in (Nadeem, 2010a) and (Nadeem, 2010b) have the form $A(V,\lambda_i) = a\cdot\exp(c\cdot V) + b\cdot\exp(d\cdot V)$, where the constants a, b, c, and d are trimmed to obtain the minimal mean square error, MSE, for the available set of experimentally determined data.

The organization of the paper is as follows. In Section 2, we introduce a model based on a standard polynomial interpolator that solves the $A(V,\lambda)$ problem and discuss its limitations. This model serves in the first place for comparison with the main model. In Section 3, we introduce a model based on fuzzy logic systems and compare that model with the one presented in Section 2. In the last section, we derive the conclusions.

2. THE POLYNOMIAL INTERPOLATOR MODEL

We first test a polynomial model (interpolator) with the expression

$$A(V,\lambda) = a(\lambda)\cdot\exp(b(\lambda)\cdot V) + c(\lambda)\cdot\exp(d(\lambda)\cdot V) \tag{10}$$

where the functions $a(\lambda), b(\lambda), c(\lambda)$ and $d(\lambda)$ have the polynomial form

$$a(\lambda) = a_0 + a_1\cdot\lambda + a_2\cdot\lambda^2 + \cdots + a_{p-1}\cdot\lambda^{p-1} \tag{11}$$

with p equal to the number of the available data sets, that is, equal to the number of available λ-independent models. Then, we obtain a system of linear equations:

$$\begin{aligned} a(\lambda_i) &= a_0 + a_1 \cdot \lambda_i + a_2 \cdot \lambda_i^2 + \cdots + a_{p-1} \cdot \lambda_i^{p-1} \\ b(\lambda_i) &= b_0 + b_1 \cdot \lambda_i + b_2 \cdot \lambda_i^2 + \cdots + b_{p-1} \cdot \lambda_i^{p-1} \\ c(\lambda_i) &= c_0 + c_1 \cdot \lambda_i + c_2 \cdot \lambda_i^2 + \cdots + c_{p-1} \cdot \lambda_i^{p-1} \\ d(\lambda_i) &= d_0 + d_1 \cdot \lambda_i + d_2 \cdot \lambda_i^2 + \cdots + d_{p-1} \cdot \lambda_i^{p-1}, \quad i = 1, \ldots p \end{aligned} \tag{12}$$

whose solution fully determines the interpolator. Based on (Nadeem, 2010a) and (Nadeem, 2010b) the solution of the above system is given in Table 1.

While testing the polynomial version of approximation for the function $A(V, \lambda)$, which is a popular approximator, we do it for sake of completeness of this study and in view of comparison with more elaborate approximators.

It is known that polynomial interpolators used as approximators have several limitations. A disadvantage of such a model is that for any new set of experimental data, the whole system of equations has to be solved again, with an increased order of the polynomial functions $a(\lambda)$, $b(\lambda)$, $c(\lambda)$, and $d(\lambda)$. The same is true whenever a set of data for a specified λ is updated by recording new data in the set. Another disadvantage of the polynomial interpolator is that there is no guarantee that the predicted attenuation functions $A(V, \lambda')$, for specified values of the wavelength, λ', vary in a reasonable range between two successive λ_i, λ_{i+1} values where experimental data are available, $\lambda_i < \lambda' < \lambda_{i+1}$. This limitation is specific to polynomial interpolators, but it is not shared by the second model presented in the next section. Finally, the polynomial interpolators are very sensitive to the values of the coefficients. The large number of decimals shown in Table 1 is required for interpolating the model in (Nadeem, 2010a) and (Nadeem, 2010b); small errors in the coefficient values in Table 1 may lead to large departures from the interpolated values. In Figure 1, we exemplify the polynomial interpolator for the parameter a in the model due to (Nadeem, 2010a), (Nadeem, 2010b). The interpolating function $a(\lambda)$ has a huge variation range between 950 and 1550 nm, leading to erroneously large attenuations.

Table 1

Coefficient values for the third order polynomial interpolator for the coefficients in the $A(V,\lambda)$ model based on (Nadeem, 2010a) and (Nadeem, 2010b)

λ_i[nm]	$a(\lambda_i)$	$b(\lambda_i)$	$c(\lambda_i)$	$d(\lambda_i)$
830	142.5	-0.004006	40.94	0.0002315
850	946.8	-0.02271	170	-0.00002916
950	733	-0.02824	130.6	-0.003764
1550	104.9	-0.005618	54.4	-0.000302

Table 1 (continued)

a_0	-612229.464798280
a_1	1766.327619378310
a_2	-1.647853128307
a_3	0.000492361772

b_0	12.64720186441800000
b_1	-0.03626764316137570
b_2	0.00003363117804233
b_3	-0.00000000999949735

c_0	-99038.543642526400
c_1	285.918794477513
c_2	-0.266873019180
c_3	0.000079777447

d_0	-0.3778649863409390
d_1	0.0011719885008267
d_2	-0.0000011677873684
d_3	0.0000000003669806

Notice that other popular interpolators, like the rational Lagrange polynomials have the same drawbacks as the polynomial interpolators. Low-order spline functions do not have uncontrolled variations between the interpolation points, yet they retain the undesirable high sensitivity to the values of the coefficients.

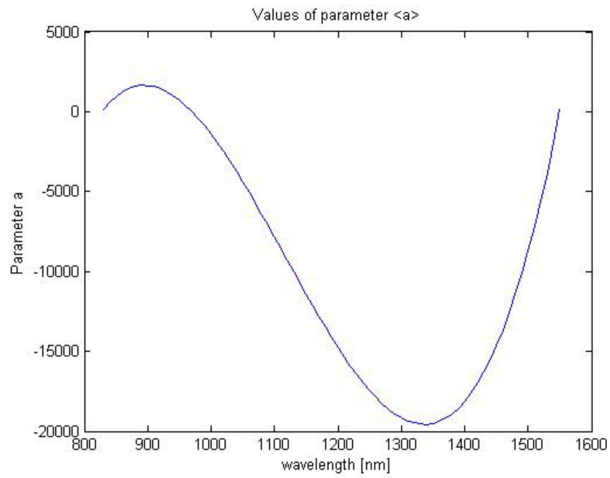

Fig. 1. Exact interpolator for the a values. Notice the erroneously huge values of the parameter for wavelengths in the range 1000–1500 nm.

Note: There is a typographical error in (Nadeem, 2010a), namely $b(\lambda) = -2.239\cdot10^{-6} - 0.002148\lambda$ in equation (14); the correct formula should be $b(\lambda) = -0.002148 - 2.239\cdot10^{-6}\lambda$. We used the corrected formula throughout this paper.

3. THE FUZZY INTERPOLATOR MODEL

The fuzzy model proposed in this section eliminated the second disadvantage of the previous model and makes the updating of the model a simple task. The fuzzy model is built as follows. First, the wavelength space is fuzzified. Triangular membership functions are used, with the upper vertex of the triangle placed in the λ_k values where experimental data are available, like in Figure 2. Notice that the triangles are neither isosceles, nor equal.

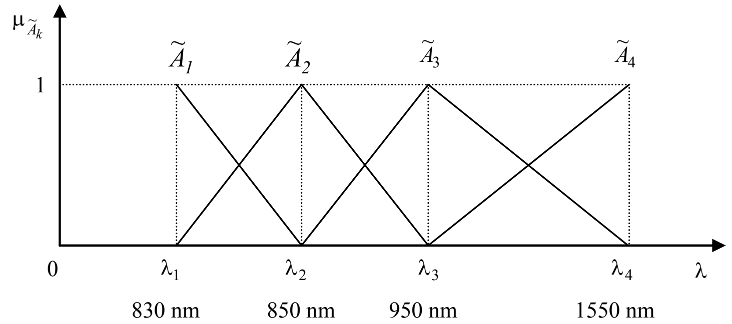

Fig. 2. Choice of input membership functions based on available data, (Henninger, 2010), (Carruthers, 2002). (Not at scale on the wavelength axis).

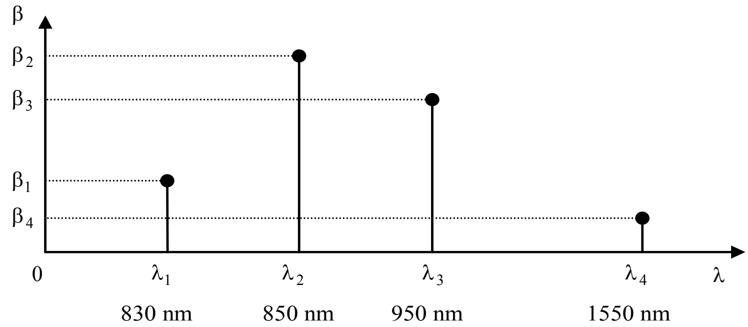

Fig. 3. Choice of the positions of the output singletons for use in equation (14).

Then, four Sugeno-type, order 0, fuzzy logic systems are defined by the following sets of rules:

System #1: *If the wavelength* λ *is* $\tilde{A}_k$*, then the coefficient a is* a_k.

System #2: *If the wavelength* λ *is* $\tilde{A}_k$*, then the coefficient b is* b_k.

System #3: *If the wavelength* λ *is* $\tilde{A}_k$*, then the coefficient c is* c_k.

System #4: *If the wavelength* λ *is* $\tilde{A}_k$*, then the coefficient d is* d_k.

The triangular membership functions are defined by the expression.

$$\mu_1(\lambda) = \begin{cases} 1 - \dfrac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} & \lambda_1 \le \lambda \le \lambda_2 \\ 0 & elsewhere \end{cases}$$

$$\mu_k(\lambda) = \begin{cases} \dfrac{\lambda - \lambda_{k-1}}{\lambda_k - \lambda_{k-1}} & \lambda_{k-1} \le \lambda < \lambda_k \\ 1 - \dfrac{\lambda - \lambda_k}{\lambda_{k+1} - \lambda_k} & \lambda_k \le \lambda < \lambda_{k+1}, \quad k = 2..p-1 \\ 0 & elsewhere \end{cases}$$

$$\mu_p(\lambda) = \begin{cases} \dfrac{\lambda - \lambda_{p-1}}{\lambda_p - \lambda_{p-1}} & \lambda_{p-1} \le \lambda \le \lambda_p \\ 0 & elsewhere \end{cases} .$$

The values of the singletons in the consequents of the rules are the same as in the corresponding particular model in (Nadem, 2010a) and (Nadeem, 2010b) for the respective λ_k values. For a specified value λ', $\lambda_k < \lambda' < \lambda_{k+1}$, the values of the coefficients are determined as (Teodorescu, 1998), (Teodorescu, 1990) and (Teodorescu, 2007)

$$a(\lambda') = a_k \cdot \left(1 - \frac{\lambda' - \lambda_k}{\lambda_{k+1} - \lambda_k}\right) + a_{k+1} \cdot \frac{\lambda' - \lambda_k}{\lambda_{k+1} - \lambda_k} \tag{13}$$

and similarly for the other coefficients, for example

$$d(\lambda') = d_k \cdot \left(1 - \frac{\lambda' - \lambda_k}{\lambda_{k+1} - \lambda_k}\right) + d_{k+1} \cdot \frac{\lambda' - \lambda_k}{\lambda_{k+1} - \lambda_k} \tag{14}$$

The above expression is derived in (Teodorescu, 1990) and (Teodorescu, 1998) from the general form of the defuzzified output of Sugeno fuzzy logic systems

$$y(x) = \sum_{i=1}^{n} \mu_{\tilde{A}_i}(x) \cdot \beta_i \Big/ \sum_{i=1}^{n} \mu_{\tilde{A}_i}(x) \tag{15}$$

where $\tilde{A}_i$ denote the input fuzzy sets, and $\beta_i \in \mathbf{R}$ are the corresponding singletons (Fig. 3). It is easy to see that for $\lambda' = \lambda_k$ one obtains the precise values reported in

(Nadeem, 2010a) and (Nadeem, 2010b). With these preparations, the attenuation model based on fuzzy logic systems is

If the wavelength λ *is* $\tilde{A}_k$*, then the attenuation is*

$$A(V,\lambda') = a(\lambda')\cdot \exp(c(\lambda')\cdot V) + b(\lambda')\cdot \exp(d(\lambda')\cdot V),$$

with the coefficients determined as above (13), (14).

The fuzzy model is easy to change when a new set of data becomes available for some new value of λ', $\lambda_k < \lambda' < \lambda_{k+1}$. Indeed, the new value λ' is inserted at the right place, a new membership function $\tilde{A}_k$ is inserted, the two neighboring membership functions are modified as shown in Figure 4, and the indices are re-written accordingly. Thus, the fuzzy model is flexible and easily changed to fit new data.

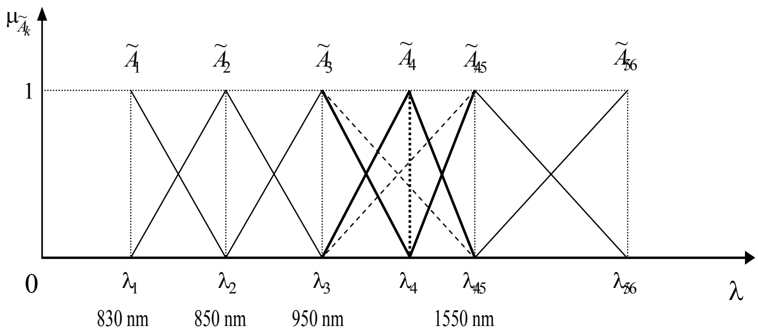

Fig. 4. Updating the fuzzy system when new data becomes available for a new value of the wavelength. (Not at scale on the wavelength axis).

Table 2

Values of the singletons for *a*, *b*, *c*, and *d*, based on (Nadem, 2010a) and (Nadeem, 2010b), and examples of approximated values of the coefficients for 840, 900, and 1250 nm

λ_k [nm]	$a_k(\lambda)$	$b_k(\lambda)$	$c_k(\lambda)$	$d_k(\lambda)$
830	142.5	-0.00401	40.94	0.0002315
850	946.8	-0.02271	170	-2.916E-05
950	733	-0.02824	130.6	-0.003764
1550	104.9	-0.00562	54.4	-0.000302

λ' [nm]	$a(\lambda')$	$b_k(\lambda')$	$c(\lambda')$	$d_k(\lambda')$
840	544.65	-0.01336	105.47	0.00010117
900	839.9	-0.02548	150.3	-0.0018966
1250	418.95	-0.01693	92.5	-0.002033

Numerical values are easily obtained from the above described fuzzy system. For example, for $\lambda' = 840\,nm$, using the coefficient values provided in (Nadem, 2010a) and (Nadeem, 2010b) for $\lambda_k = 830\,nm$ and $\lambda_{k+1} = 850\,nm$, one obtains from (13) that $a(\lambda') = 554.65$, and similarly $b(\lambda') = -0.01336$, $c(\lambda') = 105{,}47$, $d(\lambda') = 0.00010117$. In Table 2, based on the values of the coefficients *a*, *b*, *c*, and *d* from (Nadem, 2010a) and (Nadeem, 2010b), for the wavelengths 830, 850, 950 and 1550 nm, we provide the approximate values of the coefficients for the wavelengths 840, 900 and 1250 nm, as provided by the fuzzy approximator.

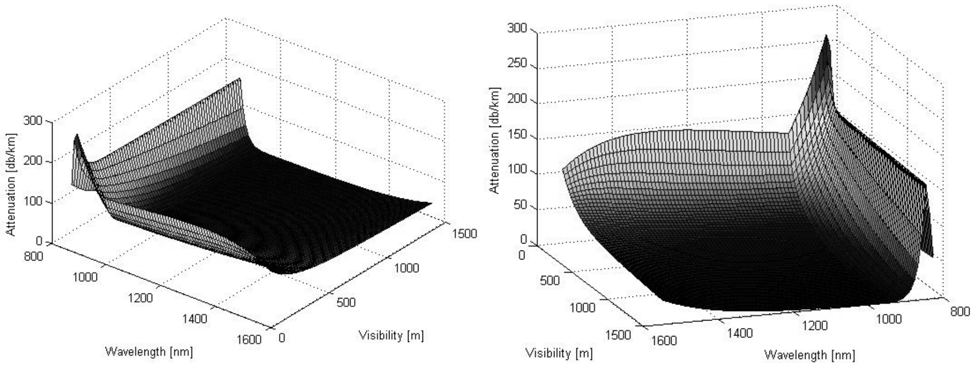

Fig. 5. Surface corresponding to the fuzzy approximator for $A(V,\lambda')$.

Notice that the function produced by the fuzzy logic system is an exact (functional) interpolator through the four functions provided by the model due to Nadeem *et al.*, and an approximator between them. The surface corresponding to the function thus generated is shown in two different views in Figure 5. Notice that the surface is smooth everywhere except in the points where it is a precise interpolator. In those points, the function is continuous, but its derivatives are not.

4. DISCUSSION AND CONCLUSIONS

We have introduced a flexible fuzzy interpolator for the fog attenuation function with respect to the wavelength for IR communications. The interpolator, which is based on the double exponential model due to Nadeem *et al.*, allows us an easy updating when new data becomes available. The fuzzy model for the attenuation function $A(V,\lambda)$ is simple enough to implement it in a spreadsheet. Yet, as other fuzzy models, it is robust to experimental data errors and to computation precision; moreover, the model does not produce the type of errors related to uncontrollable variations between the interpolation points, as seen for polynomial interpolators.

The work reported in this paper should be extended to include the contribution of wind turbulence on IR attenuation, as presented in (Pesek, 2010). Also, based on the approximate fuzzy model, formulas for solving the problem of the reliability of IR communication links, as sketched in (Takayama, 2010), remain to be derived.

Note on the related software. The Matlab implementation of the proposed model is registered with Romanian Copyright Office (ORDA) and is available from the authors, for research purposes.

Authors' contribution: SB has implemented in Matlab the polynomial and fuzzy interpolator models and contributed writing the paper; CN has tested the polynomial and fuzzy interpolator models and contributed writing the paper. HNT has proposed the models and contributed largely to writing the paper. The authors consider that they equally contributed to the paper. All the authors have seen the last form of the paper and agreed with the publication. The authors declare no conflicts of interest.

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