A SIMPLE FUZZY CONTROL DESIGN FOR POWERTRAIN SYSTEMS WITH THREE INERTIAS

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This paper proposes a simple fuzzy control design for powertrain systems with three inertias. Considering four operating modes of the clutch, the piecewise affine state-space models offers an accurate characterization of the powertrain systems as controlled processes. A class of Takagi-Sugeno fuzzy controllers (T-S FCs) is offered with this regard. The inputs of the T-S FCs are the two variables that define the four operating regimes and they also define a partition of the fifth order state-space model, the control error and the increment of control error. The control error and the increment of control error are usually involved in structures of Mamdani and Takagi-Sugeno PI-fuzzy controllers. The control error is defined considering the wheel speed as the controlled output. Neglecting the affine terms in the piecewise affine state-space models of the process, the frequency domain design is applied to the process models in terms of neglecting the affine terms to obtain four continuous-time PI controllers. The continuous-time PI controllers are next discretized and included in the six rule consequents of T-S FCs. This modal equivalence principle-based design approach offers T-S FCs that exhibit the bumpless interpolation between separately designed linear PI controllers for each operating mode of the clutch. Digital simulation results are presented to illustrate the performance of the fuzzy control system and to compare it with the linear PI-based control system that are successfully used as wheel speed control systems.

Key words: piecewise affine state-space models, PI controllers, powertrain systems with three inertias, Takagi-Sugeno fuzzy controllers.

1. INTRODUCTION

The automotive control systems are one of the key elements for innovation in vehicle industry. Since increasing overall vehicle performance specifications are...
imposed in more and more sophisticated vehicle structures, fuel economy and safety, the design of control systems for such mechatronics applications is challenging [16, 19].

In the framework of automotive applications, the powertrain systems are important systems because of the need for increased driveability and passenger comfort. The powertrain systems can be viewed as control systems themselves or as subsystems in more complex control systems [16, 19]. Some recent control approaches that deal with the damping of driveline oscillations in powertrain control systems are robust pole placement [36], linear quadratic Gaussian control with loop transfer recovery [2], model predictive control [5–10, 35], and pole placement with internal model control [28].

Using the piecewise affine state-space models of powertrain systems with three inertias given in [10] and considered as controlled processes and the excellent control results reported in [5–10] by the team of the Department of Automatic Control and Applied Informatics, “Gheorghe Asachi” Technical University of Iasi, Romania, the main contribution of this paper is a simple fuzzy control design for this process. A class of Takagi-Sugeno fuzzy controllers (T-S FCs) for the wheel speed control is suggested aiming the control system performance improvement. Our new results are important because of the simplicity of T-S FC structure characterized by only six rules and by the control design approach. The T-S FC control design is based on the separate frequency domain design of four continuous-time PI controllers that correspond to the four operating modes of the clutch in terms of neglecting the affine terms in the process models. The PI controllers are discretized and inserted in the rule consequents of T-S FCs which consist of only six rules. Therefore, the bumpless interpolation between four linear PI controllers is carried out by the T-S FCs. The design is thus based on the modal equivalence principle [13], and the suggested class of T-S FCs is a special case of PI-fuzzy controllers with several designs and real-world control applications presented in [30, 32–34]. This also outlines another importance of our approach as the nonlinearity of the fuzzy controllers can be further exploited in order to obtain the performance improvement.

The paper is organized as follows. The process model is presented in the next section. The T-S FCs and their design approach are presented in Section 3. The simulation results given in Section 4 highlight the performance improvement of the fuzzy control system compared to one of the PI controller-based linear control systems. The conclusions are drawn in Section 5.

2. PROCESS MODEL

The powertrain system with three inertias illustrated in Figure 1 consists of several subsystems [10]: the engine, the clutch, the transmission (the gear box), the final drive, the driveshaft and the wheels. The piecewise affine state-space model is
\[
\dot{x}(t) = A_i x(t) + b u(t) + f_i, \\
y(t) = c^T x(t), \quad x(t) \in \Omega_i, \quad i = 1 \ldots 4,
\]

where \( u \) is the control signal, \( u = T_e \), \( T_e \) is the torque applied to the engine, the state vector \( x(t) \) is

\[
x(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T \\
= [\theta_e(t) - \theta_i(t)i_t - \theta_w(t) - \omega_e(t) \quad \omega_e(t) \quad \omega_w(t)]^T \in \mathbb{R}^5,
\]

\( \theta_e(t) \) is the engine angle (angular position), \( \theta_i(t) \) is the transmission ratio, \( \theta_w(t) \) is the wheel angle, \( i_f \) is the final reduction gear ratio, \( \omega_e(t) \) is the engine angular speed, \( \omega_i(t) \) is the transmission angular speed, and \( \omega_w(t) \) is the wheel (angular) speed.

The subscript \( i, \quad i = 1 \ldots 4, \) in (1) indicates the index of the active mode of the clutch at the time moment \( t, \quad t \in \mathbb{R}_+ \). The affine terms \( f_i \in \mathbb{R}^{5i}, \quad i = 1 \ldots 4, \) in (1) are different for the four regions of the state-space \( \Omega_i, \quad i = 1 \ldots 4, \) that correspond to the operating modes of the clutch:

\[
\begin{align*}
\Omega_1 &= \{ x \in \mathbb{R}^5 \mid x_3 \leq \omega_e^{closing} \}, \\
\Omega_2 &= \{ x \in \mathbb{R}^5 \mid x_3 > \omega_e^{closing} \quad \text{and} \quad x_1 \leq \theta_1 \}, \\
\Omega_3 &= \{ x \in \mathbb{R}^5 \mid x_3 > \omega_e^{closing} \quad \text{and} \quad \theta_1 < x_1 \leq \theta_2 \}, \\
\Omega_4 &= \{ x \in \mathbb{R}^5 \mid x_3 > \omega_e^{closing} \quad \text{and} \quad \theta_2 < x_1 \leq \theta_3 \},
\end{align*}
\]

where \( \omega_e^{closing} \) is the engine closing speed, and \( \theta_i, \quad i = 1 \ldots 3, \) are the threshold values for torsion angle between the engine and the transmission, which are used to pass between different operating modes. A reset condition is imposed in [10] to solve the transition from the open mode to the closing mode.

The variable \( y \) in (1) is the controlled output, \( y = \omega_w \), the expression of \( c^T \) in wheel speed control systems is

\[
c^T = [0 \quad 0 \quad 0 \quad 0 \quad 1].
\]

The values of the parameters and the expressions of the other matrices specific to the model are defined in [10] and also presented in different versions in [5–10, 35].
As shown in [31], the models defined in (1) can be transformed into Linear Parameter-Varying (LPV) dynamic models by neglecting the affine terms $f_i$, and this will be used in the sequel in the T-S FC design. The Tensor Product model transformation [1, 11, 24, 29] offers a convenient way to next uniformly transform the LPV dynamic models into convex parameter-varying weighted combinations of Linear Time-Invariant systems, and appropriate control design approaches can be used. These transformations are rather general and they allow the design of linear and nonlinear controllers including fuzzy ones for a wide area of applications [3, 12, 17, 18, 21, 23, 37, 39, 41].

The control systems for the process (1) should fulfill the constraints [6, 8, 10]:

$$0 \leq u(t) \leq T_e^{\text{max}}, \quad \forall t \in \mathbb{R}_+,$$

$$T_e^{\text{m}} \leq \dot{u}(t) \leq T_e^{\text{M}}, \quad \forall t \in \mathbb{R}_+,$$

$$\omega_e^{\text{min}} \leq \omega_e(t) \leq \omega_e^{\text{max}}, \quad \forall t \in \mathbb{R}_+,$$

$$\omega_w^{\text{min}} \leq \omega_w(t) \leq \omega_w^{\text{max}}, \quad \forall t \in \mathbb{R}_+,$$

where $T_e^{\text{max}}$ is the maximum torque that can be generated by the engine, $T_e^{\text{m}}$ and $T_e^{\text{M}}$ are the torque rate bounds, $\omega_e^{\text{min}}$ and $\omega_e^{\text{max}}$ are the idle speed and the engine limit speed, respectively, and $\omega_w^{\text{min}}$ and $\omega_w^{\text{max}}$ are the minimum and the maximum angular speed of the wheels, respectively. These constraints will not be accounted for in the simple fuzzy controller design presented in the next section.
3. A CLASS OF TAKAGI-SUGENO FUZZY CONTROLLERS AND THEIR DESIGN APPROACH

The process model given in the previous section justifies the use of PI controllers to obtain acceptable control system performance (overshoot, settling time) of the speed control systems. Fuzzy controllers are designed in order to obtain the performance improvement. Therefore, the design of T-S FCs starts with the separate frequency domain design of the four linear PI controllers with the transfer functions $H_{ci}(s)$

$$H_{ci}(s) = k_{ci}(1 + s T_{ci}) / s, \quad i = 1\ldots4,$$  

(6)

where $k_{ci}, i = 1\ldots4,$ are the controller gains, $T_{ci}, i = 1\ldots4,$ are the integral time constants, and $H_{ci}(s)$ corresponds to $\Omega_i, i = 1\ldots4.$ Tustin’s method leads to the following recurrent equations of the quasi-continuous digital PI controllers obtained as the discrete time forms of (6):

$$\Delta u_k^i = K_k^i[\Delta e_k + \mu^i e_k], \quad i = 1\ldots4,$$  

(7)

where $\Delta u_k^i = u_k - u_{k-1}$ is the increment of control signal, $\Delta e_k = e_k - e_{k-1}$ is the increment of control error, $k, k \in Z_+,$ is the index of current sampling interval,

$$K_k^i = k_{ci}(T_{ci} - T_s / 2), \quad \mu^i = 2T_s / (2T_{ci} - T_s), \quad i = 1\ldots4,$$  

(8)

and $T_s$ is the sampling period.

The structure of the proposed class of T-S FCs is presented in Figure 2, where $r$ is the reference input, i.e., the desired wheel speed, $e = r - y$ is the control error, and FC is the nonlinear part (without dynamics) of T-S FC. The input membership functions of variables $x_{3,k}$ and $|x_{1,k}|$ involved in the selection of $\Omega_i, i = 1\ldots4,$ are given in Figure 3.

Fig. 2. Structure of Takagi-Sugeno fuzzy controllers.
The other two input variables, $e_k$ and $\Delta e_k$, are employed as inputs in (7), and they are placed in the rule consequents of the T-S FCs. A single linguistic term is used for these inputs, and the fuzzy set that corresponds to these inputs are universal; that is the reason why the membership functions of $e_k$ and $\Delta e_k$ are not included in Figure 3. Figure 3 points out the three tuning parameters of the antecedent part of T-S FCs, i.e., $\omega_{x3}$, $\theta_{x1}$ and $\theta_{x2}$. The parameters of the consequent part of T-S FCs are $k_{e,i}$, $i = 1...4$, $T_{e,i}$, $i = 1...4$, and $T_s$. Other parameters can be defined in the inference engine and in the defuzzification method. Our class of T-S FCs uses the SUM and PROD operators in the inference engine and the weighted sum defuzzification method.

![Fig. 3. Membership functions of first two input variables.](image)

The inference engine is assisted by the rule base of T-S FC that consists of the six rules $R^1, R^2, ..., R^6$

\[
R^1: \text{IF } x_{3,k} \text{ IS PS AND } |x_{1,k}| \text{ IS PS THEN } \Delta u_k = \Delta u^1_k, \\
R^2: \text{IF } x_{3,k} \text{ IS PS AND } |x_{1,k}| \text{ IS PM THEN } \Delta u_k = \Delta u^2_k, \\
R^3: \text{IF } x_{3,k} \text{ IS PS AND } |x_{1,k}| \text{ IS PB THEN } \Delta u_k = \Delta u^3_k, \\
R^4: \text{IF } x_{3,k} \text{ IS PB AND } |x_{1,k}| \text{ IS PS THEN } \Delta u_k = \Delta u^4_k, \\
R^5: \text{IF } x_{3,k} \text{ IS PB AND } |x_{1,k}| \text{ IS PM THEN } \Delta u_k = \Delta u^5_k, \\
R^6: \text{IF } x_{3,k} \text{ IS PB AND } |x_{1,k}| \text{ IS PB THEN } \Delta u_k = \Delta u^6_k.
\]

(9)

Additional membership functions can be defined for $e_k$ and $\Delta e_k$ as in [30, 32–34], for the sake of fuzzy control system performance improvement. However, this affects the simple design aimed in this paper as it increases the implementation and design costs because the rule base is more complicated and more parameters are defined.
Our design approach, dedicated to the previously defined class of T-S FCs consists of the following design steps.

**Step 1.** Apply the frequency domain design to tune the parameters of the PI controllers with the transfer functions given in (6) for the process model (1) with \( f_i = 0 \), \( i = 1...4 \).

**Step 2.** Set the sampling period \( T_s \) according to the requirements of quasi-continuous digital control and apply (8) to obtain the rule consequent parameters in (9).

**Step 3.** Choose the values of the parameters \( \omega_{c3} \), \( \theta_{s1} \) and \( \theta_{s2} \) such that to obtain the firing of all linguistic terms presented in Fig. 3 and of all six rules for the important operating regimes of powertrain systems.

### 4. SIMULATION RESULTS

This section presents the results of the application of the design approach presented in the previous section in the design of a T-S PI-FC for the wheel speed control. Using the process parameter values given in [10], step 1 of the design approach makes use of the following matrices in the process model (1) for \( f_i = 0 \), \( i = 1...4 \):

\[
A_i = [a^{i}_{mn}]_{m,n=1...5}, \quad i = 1...4,
\]

\[
a_{i1}^{1} = a_{i2}^{1} = a_{i4}^{1} = a_{i5}^{1} = a_{i2}^{2} = a_{i3}^{2} = a_{i4}^{2} = a_{i5}^{2} = 0, \quad i = 1...4,
\]

\[
a_{i3}^{1} = 1, a_{i4}^{1} = -3.5, a_{i4}^{2} = 0.2703, a_{i5}^{2} = -1, a_{i4}^{3} = -0.8309 \cdot 10^5, \quad i = 1...4,
\]

\[
a_{i5}^{2} = 1080.1222, a_{i3}^{3} = 34.6356, a_{i4}^{3} = -0.1217, a_{i5}^{3} = -0.4535, \quad i = 1...4,
\]

\[
a_{i3}^{4} = 0, a_{i3}^{5} = -4704.8824, a_{i4}^{5} = -9411.7647, a_{i3}^{6} = -18823.5294
\]

\[
a_{i3}^{5} = -0.9353, a_{i3}^{6} = -18.5824, a_{i3}^{7} = -36.2294, a_{i3}^{8} = -59.7588,
\]

\[
a_{i3}^{6} = 0, a_{i3}^{7} = 61.7647, a_{i3}^{8} = 123.5294, a_{i4}^{8} = 205.8824,
\]

\[
a_{i4}^{1} = 0, a_{i4}^{2} = 1.7215 \cdot 10^5, a_{i4}^{3} = 3.4431 \cdot 10^5, a_{i4}^{4} = 6.8862 \cdot 10^5,
\]

\[
a_{i4}^{5} = -298.5224, a_{i4}^{6} = -2558.0549, a_{i4}^{7} = -4817.5873, a_{i4}^{8} = -7830.2973,
\]

\[
b = [0 \quad 0 \quad 5.8824 \quad 0]^T, \quad c^T = [0 \quad 0 \quad 0 \quad 1].
\]

Imposing the phase margin of 60°, the frequency domain design leads to the open-loop Bode plots illustrated in Figures 4 to 6 and to the PI controller parameters \( k_{c1} = 1 \), \( T_{c1} = 1 \) s, \( k_{c2} = 5 \), \( T_{c2} = 2.3 \) s, \( k_{c3} = 30 \), \( T_{c3} = 1 \) s, \( k_{c4} = 20 \) and \( T_{c4} = 1 \) s.

Using the process parameter values given in [10]

\[
\theta_1 = 0.1745 \text{ rad}, \quad \theta_2 = 0.2094 \text{ rad}, \quad \omega_e^{\text{cos}g} = 125.6637 \text{ rad/s},
\]

(11)
Fig. 4. Open-loop Bode plots for the linear control system in the region $\Omega_1$ of the state-space.

Fig. 5. Open-loop Bode plots for the linear control system in the region $\Omega_2$ of the state-space.
Fig. 6. Open-loop Bode plots for the linear control system in the region $\Omega_3$ of the state-space.

Fig. 7. Open-loop Bode plots for the linear control system in the region $\Omega_4$ of the state-space.
and setting the sampling period $T_s = 0.005 \text{s}$, equation (8) applied in step 2 of the design approach leads to the parameters of the digital PI controllers in the rule consequents $K_p^1 = 0.9975$, $\mu^1 = 0.005$, $K_p^2 = 11.4875$, $\mu^2 = 0.0022$, $K_p^3 = 29.925$, $\mu^3 = 0.005$, $K_p^4 = 19.95$ and $\mu^4 = 0.005$. The parameters values of the parameters of the input membership functions are chosen as follows in step 3:

$$\theta_{s1} = 0.01 \text{ rad}, \theta_{s2} = 0.01 \text{ rad}, \omega_{s3} = 10 \text{ rad/s}.$$  \tag{12}$$

The simulation of the fuzzy control system with this set of parameters for the $r = 40 \text{ rad/s}$ step modification of the reference input followed by an additive 4 rad/s disturbance input applied to the process output at the time moment of 20 s leads to the system responses presented in Figure 8.

![Fig. 8. Simulation results for the fuzzy control system.](image)

The behavior of the fuzzy control system is compared to that of a linear control system with one of the PI controllers, viz. the PI controller with the transfer function $H_{c1}(s)$. The simulation results for the linear control system are presented in Figure 9.
The simulation results presented in Figures 8 and 9 show the control system performance improvement ensured by the T-S FC in regulation and tracking. This improvement concerns both the settling time and the overshoot.

Although those constraints were not used in the design, the constraints in [6, 8, 10] are fulfilled. The inclusion of constraints in the design can be achieved conveniently by the optimal tuning of T-S FCs. Appropriate optimization problems should be defined with this regard, and several optimization approaches and algorithms can be applied [4, 15, 22, 26, 27, 30, 38] focusing on the additional performance improvement. Other stability constraints expressed as linear matrix inequalities can be introduced and solved in the framework of parallel distributed compensation [14, 20, 25, 40].

5. CONCLUSIONS

This paper has proposed a six-step design approach for a class of T-S FCs dedicated to powertrain systems with three inertias. The suggested that a class of T-S FCs ensures the bumpless interpolation between four separately designed
PI controllers. The design approach has been successfully applied in the design of a wheel speed controller, and the digital simulation results prove the performance improvement of the fuzzy control system with respect to the linear control system.

The main limitation of our approach is the need to choose three parameters that characterize the input membership functions. Future research aims the convenient solving of this problem by the stable design of fuzzy controllers combined with the definition or appropriate optimization problems to conduct the optimal parameter tuning of fuzzy controllers as well. More complicated controller structures will be considered.

Acknowledgments. This work was supported by the National Center for Programs Management from Romania, CNMP, under the research grant SICONA – 12100/2008, by a grant of the Romanian National Authority for Scientific Research, CNCS – UEFISCDI, project number PN-II-ID-PCE-2011-3-0109, and by a grant from the NSERC of Canada. The authors express many thanks to their colleagues from the Department of Automatic Control and Applied Informatics, “Gheorghe Asachi” Technical University of Iasi, Romania, for their fruitful co-operation and discussions concerning the models and for the generous providing of several models and control results as well.

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Received May 4, 2013