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# L - Substructures of L-Subsets

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**Abstract:** In this paper, we have introduced the concept of L-*subsets* of a group *satisfying* the inverse *property* and the concept of L-*subgroups* of an L-*subset which* satisfies the inverse *property*, where L is a lattice.

**Key word:** L-subgroups, L-subgroups of L-subsets.

## 1. Introduction

Throughout this paper, unless otherwise stated, $L$ always denotes a *distributive lattice*.

A *complete Heyting algebra* [2] $A$ is a *complete lattice* such that for all $B \subseteq A$ and for all $b \in A$, $\vee\{a \wedge b: a \in B\} = [\vee\{a: a \in B\}] \wedge b$ and

$$\wedge\{a \vee b: a \in B\} = [\wedge\{a: a \in B\}] \vee b.$$

We at times assume that $L$ is a *complete Heyting algebra* or a *chain*. An *L-subset* [1] of a nonempty *set* $X$ is a *function* from $X$ into $L$. The *set* of all *L-subsets* of a nonempty *set* $X$ is called the *L-power set* of $X$ and is denoted by $L^X$.

Throughout this paper $G$ always denotes an arbitrary group with a multiplicative *binary operation which* we shall denote by juxtaposition. The multiplicative *identity* of $G$ shall be denoted by $e$.

**Definition 1.1**[2]. An *L- subset* $\mu$ of $G$ is called an *L - subgroup* of $G$ if

(1.1) $\mu(xy) \geq \mu(x) \wedge \mu(y)$ $\forall$ $x, y \in G$, and

(1.2) $\mu(x^{-1}) \geq \mu(x)$ $\forall$ $x \in G$.

If $L = [0, 1]$, then an *L - subgroup of G* is called a fuzzy *subgroup of G*. The set of all *L - subgroups* of $G$ shall be denoted by $L(G)$ and the *set* of all fuzzy *subgroups* of $G$ shall be denoted by $I(G)$.

**Lemma 1.1** [2]. Let $\mu \in L(G)$. Then $\forall$ $x \in G$,

(1.3) $\mu(e) \geq \mu(x)$ ;

(1.4) $\mu(x^{-1}) = \mu(x)$.

Let $L$ be a *complete lattice* and $\mu$, $\sigma \in L^G$.

The *product* [2] of $\mu$ and $\sigma$, denoted by $\mu o \sigma$, is defined as follows:

$\forall$ $x \in G$,

(1.5) $(\mu o \sigma)(x) = \vee\{\mu(y) \wedge \sigma(z)$: $y, z \in G, x = yz\}$.
Let $\mu \in L^G$. The inverse of $\mu$ [2], denoted by $\mu^{-1}$, is defined as follows:
$\forall\ x \in G$,
(1.6) $\mu^{-1}(x) = \mu(x^{-1})$.
It is clear that $\forall\ \mu, \sigma \in L^G$, $\mu o \sigma \in L^G$ and $\mu^{-1} \in L^G$.

**Theorem 1.2** [2]. Let $\mu \in L^G$. Then $\mu$ is an *L-subgroup* of $G$ if and only if $\mu_\alpha$ is a *subgroup* of $G$, $\forall\ \alpha \in \text{Im}(\mu) \cup \{b \in L$: $b \leq \mu(e)\}$.

**Theorem 1.3** [2]. Let $\mu \in L^G$. Then $\mu \in L(G)$ if and only if $\mu$ satisfies the following conditions:
(1.7) $\mu o \mu \subseteq \mu$;
(1.8) $\mu^{-1} \subseteq \mu$ ( or $\mu \subseteq \mu^{-1}$, or $\mu^{-1} = \mu$ ).

**Theorem 1.4** [2]. Let $\mu, \nu \in L(G)$. Then $\mu o \nu \in L(G)$ if and only if $\mu o \nu = \nu o \mu$.

## 2. L – Subgroups of L – Subsets

In this section, we introduce two definitions, one is *L-subset* of $G$ satisfying the *inverse property* and the other is *L-subgroup* of an *L-subset* of $G$ that satisfies the *inverse property.* Then we prove some fundamental results thereof.

**Definition 2.1.** Let $\mu \in L^G$. We say that $\mu$ satisfies the *inverse property* if $\mu(x) = \mu(x^{-1})\ \forall x \in G$.

We observe that if $\mu \in L(G)$, then $\mu$ satisfies the *inverse property*. We also observe that if $\mu \in L^G$, then $\mu$ satisfies the *inverse property* if and only if

$$\mu = \mu^{-1}.$$

The *set* of all *L-subsets* of $G$, which satisfy the inverse property, shall be denoted by $LS(G)$. Note that $\mu \in L(G)$ implies $\mu \in LS(G)$.

**Definition 2.2.** Let $\mu \in L^G$, $\nu \in LS(G)$ and $\mu \subseteq \nu$. Then we say that $\mu$ is an *L*-subgroup of ν if $\forall\ x, y \in G$,

$$\mu(xy) \geq \mu(x) \wedge \nu(y).$$

Let $\nu \in LS(G)$. The *set* of all *L-subgroups* of $\nu$ shall be denoted by $LS_G(\nu)$. If we write $\mu \in LS_G(\nu)$, it is always understood that $G$ is a *group*,

$$\mu, \nu \in L^G,\ \nu \in LS(G), \mu \subseteq \nu,$$

and μ is an *L-subgroup* of $\nu$.

**Example 2.1.** Example of an $\mu \in L^G$ and $\nu \in LS(G)$ such that $\mu \in LS_G(\nu)$.
Let $G = \{ e, x, x^2, x^3 \}$, $x^4 = e$, and $L = [0, 1]$.
We define $\mu$, $\nu \in L^G$ as follows:

$$\nu = \{(e, 1), (x,.3), (x^2,.6), (x^3,.3)\}, \text{ and}$$

$$\mu = \{(e,.4), (x,.3), (x^2,.4), (x^3,.3)\}.$$

Then $\nu \in LS(G)$ and $\mu \subseteq \nu$ and $\mu \in LS_G(\nu)$.

**Proposition 2.2.** $\mu \in L(G)$ implies $\mu \in LS_G(\mu)$ $\forall$ $\mu \in L^G$.
**Proposition 2.2.** Let $\mu \in L^G$, $\nu \in LS(G)$ and $\mu \subseteq \nu$. If $\mu \in LS_G(\nu)$ then

$$\mu(e) \geq \mu(x) \ \forall \ x \in G.$$

**Theorem 2.3.** Let $\mu \in L^G$, $\nu \in LS(G)$, and $\mu \subseteq \nu$. If $\mu \in LS_G(\nu)$, then $\mu \in L(G)$.
The following example shows that the converse of Theorem 2.3 is not necessarily true.

**Example 2.2.** Let $G = \{ e, x, x^2, x^3 \}$, $x^4 = e$. Suppose $L$ is a *chain*. We define $\mu$, $\nu \in L^G$ as follows**:**

$$\nu = \{ (e, p), (x, q), (x^2, r), (x^3, q) \}, \text{ where } p, q, r \in L \text{ and } p > q > r.$$

$$\mu = \{ (e, s), (x, u), (x^2, t), (x^3, u) \}, \text{ where } s, t, u \in L \text{ and } r > s > t > u.$$

We observe that $\nu \in LS(G)$ and $\mu \subseteq \nu$.
Also $\mu_s = \{e\}$, $\mu_t = \{e, x^2\}$, $\mu_u = G$ are all *subgroups* of $G$.
Consequently by Theorem 1.2, $\mu$ is an *L- subgroup* of $G$, i.e., $\mu \in L(G)$.
We find that

$$\mu(x^2 x^3) = \mu(x^5) = \mu(x) = u, \text{ and } \mu(x^2) \wedge \nu(x^3) = t \wedge q = t.$$

Since $t > u$, we have

$$\mu(x^2 x^3) \leq \mu(x^2) \wedge \nu(x^3).$$

Hence $\mu \notin LS_G(\nu)$. Thus $\mu \in L(G)$, $\nu \in LS(\nu)$, and $\mu \subseteq \nu$, whereas $\mu \notin LS_G(\nu)$.

**Theorem 2.4.** Let $\mu \in L^G$, $\nu \in LS(G)$, and $\mu \subseteq \nu$. If $\mu \in LS_G(\nu)$, then

$$\mu(x) = \mu(e) \wedge \nu(x) \ \forall \ x \in G.$$

**Proof.** Let $\mu \in LS_G(\nu)$. Then $\forall$ $x \in G$,

$$\mu(x) = \mu(ex) \geq \mu(e) \wedge \nu(x)$$

$$\geq \mu(e) \wedge \mu(x), \text{ since } \mu \subseteq \nu$$

$= \mu(x)$, using Proposition 2.2.

Hence $\mu(x) = \mu(e) \wedge \nu(x)\ \forall\ x \in G$.

**Corollary 2.5.** If $\mu$, $\sigma \in LS_G(\nu)$, then $\mu(x) \wedge \sigma(y) = \mu(y) \wedge \sigma(x)\ \forall\ x, y \in G$.

**Corollary 2.6.** If $\mu \in LS_G(\nu)$, then $\mu(x) \wedge \nu(y) = \mu(y) \wedge \nu(x)\ \forall\ x, y \in G$, and $\mu(x) \wedge \mu(y) = \mu(x) \wedge \nu(y)\ \forall\ x, y \in G$.

**Theorem 2.7.** Let $\mu \in L^G$, $\nu \in LS(G)$, and $\mu \subseteq \nu$. Then $\mu \in LS_G(\nu)$ if and only if $\mu \in L(G)$ and $\mu(x) = \mu(e) \wedge \nu(x)\ \forall\ x \in G$.

**Proof.** Let $\mu \in LS_G(\nu)$. Then by Theorem 2.3, we have $\mu \in L(G)$, and by Theorem 2.4, we have $\mu(x) = \mu(e) \wedge \nu(x)\ \forall\ x \in G$.

Conversely, suppose that $\mu \in L(G)$ and $\mu(x) = \mu(e) \wedge \nu(x)\ \forall\ x \in G$. Let $x, y \in G$. Then $\mu(xy) \geq \mu(x) \wedge \mu(y) = \mu(x) \wedge \mu(e) \wedge \nu(y) = \mu(x) \wedge \nu(y)$. Hence $\mu \in LS_G(\nu)$.

## 3. Product in $LS_G(\nu)$

In this section we obtain several results in connection with the product in $LS_G(\nu)$, where $\nu \in LS(G)$.

**Theorem 3.1.** Let L be a complete *lattice*. Then the product in $LS_G(\nu)$ is *commutative*.

**Corollary 3.2.** Let $L$ be a *complete lattice*. If $\mu$, $\sigma \in LS_G(\nu)$, then $\mu o \sigma \in L(G)$.

**Theorem 3.3.** Let $L$ be a *complete Heyting algebra*. If $\mu$, $\sigma \in LS_G(\nu)$, then $(\mu o \sigma)(x) = (\mu o \sigma)(e) \wedge (\nu o \nu)(x)\ \forall\ x \in G$.

**Proof.** Let $\mu$, $\sigma \in LS_G(\nu)$. Then by Theorem 2.4, we have

$$\mu(x) = \mu(e) \wedge \nu(x)\ \forall\ x \in G \text{ and } \sigma(x) = \sigma(e) \wedge \nu(x)\ \forall\ x \in G.$$

Now, $\forall\ x \in G$,

$$\begin{aligned}(\mu o \sigma)(x) &= \vee\{\mu(y) \wedge \sigma(z) : y, z \in G, yz = x\} \\ &= \vee\{\mu(e) \wedge \nu(y) \wedge \sigma(e) \wedge \nu(z) : y, z \in G, yz = x\} \\ &= \mu(e) \wedge \sigma(e) \wedge \{\vee\{\nu(y) \wedge \nu(z)\} : y, z \in G, yz = x\} \\ &= \mu(e) \wedge \sigma(e) \wedge (\nu o \nu)(x).\end{aligned}$$

Hence $(\mu o \sigma)(x) = \mu(e) \wedge \sigma(e) \wedge (\nu o \nu)(x)\ \forall\ x \in G$.

On the other hand, since $\mu,\ \sigma \in LS_G(\nu)$, by Proposition 2.2, we have

$$\mu(e) \geq \mu(x),\ \sigma(e) \geq \sigma(x)\ \forall\ x \in G.$$

Hence $(\mu o \sigma)(e) = \vee\{\mu(x) \wedge \sigma(x^{-1}) \colon x \in G\} = \mu(e) \wedge \sigma(e)$.
Thus $(\mu o \sigma)(x) = (\mu o \sigma)(e) \wedge (\nu o \nu)(x)\ \forall\ x \in G$.

**Theorem 3.4.** Let $L$ be a *complete Heyting algebra*. If $\mu,\ \sigma \in LS_G(\nu)$, then:

$$\mu o \sigma \in LS_G(\nu o \nu).$$

**Proof.** Using Corollary 3.2, Theorem 3.3 and Theorem 2.7, we get the result.

**Theorem 3.5.** Let $L$ be a *complete lattice*. Suppose $\mu \in L^G$, $\nu \in LS(G)$, and $\mu \subseteq \nu$. Then $\mu \in LS_G(\nu)$ if and only if $\mu o \nu \subseteq \mu$.

**Proof.** Suppose $\mu \in LS_G(\nu)$. Then $\forall\ x,\ y \in G,\ \mu(xy) \geq \mu(x) \wedge \nu(y)$.
Let $x,\ y,\ z \in G$ and $x = yz$. Then $\mu(x) = \mu(yz) \geq \mu(y) \wedge \nu(z)$, and so

$$\mu(x) \geq \vee\{\ \mu(y) \wedge \nu(z) \colon y,\ z \in G,\ yz = x\ \}$$

$$= (\mu o \nu)(x).$$

Thus $\mu(x) \geq (\mu o \nu)(x)\ \forall\ x \in G$. Hence $\mu o \nu \subseteq \mu$.

Conversely, suppose that $\mu o \nu \subseteq \mu$.

Then $\forall\ x,\ y \in G,\ \mu(xy) \geq (\mu o \nu)(xy) \geq \mu(x) \wedge \nu(y)$. Hence $\mu \in LS_G(\nu)$. This completes the proof.

**Theorem 3.6.** Let $L$ be a *complete lattice*. Suppose $\mu,\ \sigma \in L^G$, $\nu \in LS(G)$, and $\mu,\ \sigma \subseteq \nu$. If $\mu,\ \sigma \in LS_G(\nu)$ then $\mu o \sigma \in LS_G(\nu)$ ( $\sigma o \mu \in LS_G(\nu)$).

**Proof.** Let $\mu,\ \sigma \in LS_G(\nu)$. Then by theorerm 3.5, we have $\mu\ o\ \nu \subseteq \mu$ and also we have $\sigma\ o\ \nu \subseteq \sigma$. Also $\mu\ o\ \sigma \subseteq \mu\ o\ \nu \subseteq \mu \subseteq \nu$, and $(\mu\ o\ \sigma) o \nu = \mu o(\sigma\ o\ \nu) \subseteq \mu\ o\ \sigma$. Hence by Theorem 3.5, we get $\mu\ o\ \sigma \in LS_G(\nu)$. Since $\mu,\ \sigma \in LS_G(\nu)$ implies:

$\mu\ o\ \sigma = \sigma\ o\ \mu$, it follows that $\sigma\ o\ \mu \in LS_G(\nu)$.

**Theorem 3.7.** Let $L$ be a *complete lattice*.

Suppose $\mu \in L^G$, $\nu \in LS(G)$, $\nu(e) \geq \nu(x)\ \forall\ x \in G$, and $\mu \subseteq \nu$. Then $\mu \in LS_G(\nu)$ if and only if

$$\sigma\ o\ \mu \in LS_G(\nu)\ \forall\ \sigma \in L^G,\ \sigma \subseteq \nu.$$

**Proof.** Let $\mu \in LS_G(\nu)$. Then by Theorem 3.5, $\mu o \nu \subseteq \mu$. Also by Corollary 2.6, $\mu(x) \wedge \nu(y) = \mu(y) \wedge \nu(x)\ \forall\ x,\ y \in G$.

Now $\forall\ x \in G$,

$$(\mu \text{ o} \nu)(x) = \vee\{ \mu(y) \wedge \nu(z): y, z \in G, yz = x \}$$

$$= \vee\{ \mu(z) \wedge \nu(y): y, z \in G, yz = x \}$$

$$= \vee\{ \nu(y) \wedge \mu(z): y, z \in G, yz = x \}$$

$$= (\nu \text{ o } \mu)(x).$$

Thus $(\mu \text{ o } \nu)(x) = (\nu \text{ o } \mu)(x)\ \forall\ x\in G$. Hence $\mu \text{ o } \nu = \nu \text{ o } \mu$.

Let $\sigma \in L^G$ and $\sigma \subseteq \nu$. We observe that $\sigma \text{ o } \mu \subseteq \nu \text{ o } \mu = \mu \text{ o } \nu \subseteq \mu \subseteq \nu$ and

$$(\sigma\text{o}\mu)\text{o}\nu = \sigma\text{o}(\mu\text{o}\nu) \subseteq \sigma\text{o}\mu$$

Consequently by Theorem 3.5, we have $\sigma\text{o}\mu \in LS_G(\nu)$. Thus $\mu\in LS_G(\nu)$ implies $\sigma\text{o}\mu\in LS_G(\nu)\ \forall\ \sigma\in L^G,\ \sigma\subseteq \nu$.

Conversely, suppose that $\sigma\text{o}\mu \in LS_G(\nu)\ \forall\ \sigma\in L^G,\ \sigma\subseteq \nu$.

Let $\alpha = \vee\{ \mu(x): x \in G \}$ and $\sigma = e_\alpha$. Since $\nu(e) \geq \nu(x) \geq \mu(x)\ \forall\ x \in G$, it follows that $\nu(e) \geq \vee\{\mu(x): x \in G\} = \alpha$. Hence $\sigma = e_\alpha \subseteq \nu$. By our hypothesis $e_\alpha\text{o}\mu\in LS_G(\nu)$. It is easy to verify that $e_\alpha\text{o}\mu = \mu$. Hence $\mu\in LS_G(\nu)$. This completes the proof.

## 4. Cartesian Product

In this section we assume that $G$ and $H$ are two *groups* with the *identity elements* $e$ and $e'$, respectively.

Let $\nu\in LS(G)$ and $\gamma\in LS(H)$, then $\forall\ (x, h)\in G \times H$,

$$(\nu\times \gamma)(x, h)^{-1} = (\nu\times \gamma)(x^{-1}, h^{-1}) = \nu(x^{-1}) \wedge \gamma(h^{-1}) = \nu(x) \wedge \gamma(h) = (\nu\times \gamma)(x, h)$$

and so

$$\nu\times \gamma\in LS(G \times H).$$

**Theorem 4.1.** If $\mu\in LS_G(\nu)$ and $\sigma\in LS_H(\gamma)$, then $\mu \times \sigma \in LS_{G\times H}(\nu\times \gamma)$.

**Proof.** Let $\mu\in LS_G(\nu)$ and $\sigma\in LS_H(\gamma)$.
Then $\mu\in L^G$, $\nu\in LS(G)$, and $\mu \subseteq \nu$, also $\sigma\in L^H$, $\gamma\in LS(H)$, and $\sigma\subseteq \gamma$.
Thus $\mu \times \sigma\in L^{G\times H,}$ $\nu\times \gamma\in LS(G\times H)$, and $\mu \times \sigma\subseteq \nu\times \gamma$.
Then $\forall\ (x, h), (y, g)\in G \times H$,

$$(\mu\times \sigma)(x, h)(y, g) = (\mu \times \sigma)(xy, hg)$$

$$= \mu(xy) \wedge \sigma(hg)$$

$$\geq \mu(x) \wedge \nu(y) \wedge \sigma(h) \wedge \gamma(g)$$

$$= \mu(x) \wedge \sigma(h) \wedge \nu(y) \wedge \gamma(g)$$

$$= (\mu \times \sigma)(x, h) \wedge (\nu \times \gamma)(y, g).$$

Hence $\mu \times \sigma \in LS_{G\times H}(\nu \times \gamma)$.

**Theorem 4.2.** Let $L$ be a *chain*. Suppose $\mu \in L^G$, $\nu \in LS(G)$, and $\mu \subseteq \nu$, also $\sigma \in L^H$, $\gamma \in LS(H)$, and $\sigma \subseteq \gamma$. If $\mu \times \sigma \in LS_{G\times H}(\nu \times \gamma)$, then at least one of the following statements must hold.

(4.1) $\sigma(e') \geq \mu(x)\ \forall\ x \in G$,

(4.2) $\mu(e) \geq \sigma(h)\ \forall\ h \in H$.

**Proof.** Let $\mu \times \sigma \in LS_{G\times H}(\nu \times \gamma)$.
Then by Proposition 2.2, $(\mu \times \sigma)(e, e') \geq (\mu \times \sigma)(x, h)\ \forall (x, h) \in G \times H$. By contraposition, suppose that none of the statements (4.1) and (4.2) holds. Then $L$ being a *chain*, we can find $a$ in $G$ and $b$ in $H$ such that

$$\mu(a) > \sigma(e') \text{ and } \sigma(b) > \mu(e).$$

Now we find,

$$(\mu \times \sigma)(a, b) = \mu(a) \wedge \sigma(b)$$

$$> \sigma(e') \wedge \mu(e)$$

$$= \mu(e) \wedge \sigma(e')$$

$$= (\mu \times \sigma)(e, e').$$

Which contradicts the fact that $(\mu \times \sigma)(e, e') \geq (\mu \times \sigma)(x, h)\ \forall\ (x, h) \in G \times H$. Hence either $\sigma(e') \geq \mu(x)\ \forall\ x \in G$ or $\mu(e) \geq \sigma(h)\ \forall\ h \in H$.

**Theorem 4.3.** Let $L$ be a *chain*. Suppose $\mu \in L^G$, $\nu \in LS(G)$, and $\mu \subseteq \nu$, also $\sigma \in L^H$, $\gamma \in LS(H)$, and $\sigma \subseteq \gamma$.
If $\mu \times \sigma \in LS_{G\times H}(\nu \times \gamma)$ and $\mu(x) \leq \sigma(e')\ \forall\ x \in G$, then:

$$\mu \in LS_G(\nu).$$

**Proof.** Let $\mu \times \sigma \in LS_{G\times H}(\nu \times \gamma)$, and $\mu(x) \leq \sigma(e')\ \forall\ x \in G$.
Let $x, y \in G$. Since $xy \in G$ and by hypothesis $\mu(xy) \leq \sigma(e')$, we have

$$\mu(xy) = \mu(xy) \wedge \sigma(e')$$

$$= \mu(xy) \wedge \sigma(e'\, e')$$

$= (\mu \times \sigma)(xy, e' e')$

$= (\mu \times \sigma)( (x, e') (y, e') )$

$\geq (\mu \times \sigma)(x, e') \wedge (\nu \times \gamma)(y, e')$, since $\mu \times \sigma \in LS_{G \times H} (\nu \times \gamma)$

$= \mu(x) \wedge \sigma( e' ) \wedge \nu(y) \wedge \gamma( e')$

$= \mu(x) \wedge \sigma( e' ) \wedge \gamma( e') \wedge \nu(y)$

$= \mu(x) \wedge \sigma( e' ) \wedge \nu(y)$, since $\sigma \subseteq \gamma$

$= \mu(x) \wedge \nu(y)$, since $\mu(x) \leq \sigma( e') \; \forall \; x \in G$.

Thus $\mu(xy) \geq \mu(x) \wedge \nu(y) \; \forall \; x, y \in G$. Hence $\mu \in LS_G(\nu)$.
By symmetry we can prove:

**Theorem 4.4.** Let $L$ be a *chain*. Suppose $\mu \in L^G$, $\nu \in LS(G)$, and $\mu \subseteq \nu$, also $\sigma \in L^H$, $\gamma \in LS(H)$, and $\sigma \subseteq \gamma$. If $\mu \times \sigma \in LS_{G \times H} (\nu \times \gamma)$ and $\sigma (h) \leq \mu (e) \; \forall \; h \in H$, then $\sigma \in LS_H(\gamma)$.

From Theorems 4.2, 4.3, and 4.4 we have the following Corollary.

**Corollary 4.5.** Let $L$ be a *chain*. Suppose $\mu \in L^G$, $\nu \in LS(G)$, and $\mu \subseteq \nu$, also $\sigma \in L^H$, $\gamma \in LS(H)$, and $\sigma \subseteq \gamma$. If $\mu \times \sigma \in LS_{G \times H} (\nu \times \gamma)$, then either $\mu \in LS_G(\nu)$ or $\sigma \in LS_H(\gamma)$.

## 5. Images and Inverse Images under Homomorphisms

In this section, $L$ denotes a *complete Heyting Algebra*. We assume that $G$ and $H$ be two *groups* with *identities* $e$ and $e'$, respectively.

Here we shall study the *image* and *inverse image* of *L-subgroups* of *L-subsets* under *homomorphism* of $G$ into $H$. The results are the consequence of the following two theorems.

**Theorem 5.1** [2]. Let $\mu \in L(G)$. Suppose $f$ is an *epimorphism* of $G$ onto $H$. Then $f(\mu) \in L(H)$.

**Theorem 5.2** [2]. Let $\nu \in L(H)$. If $f$ is a *homomorphism* of $G$ into $H$, then $f^{-1}(\nu) \in L(G)$.

**Theorem 5.3.** Let $\mu \in L^G$ and $\nu \in LS(G)$ and $\mu \subseteq \nu$. If $\mu \in LS_G(\nu)$ and $f$ is an *epimorphism* of $G$ onto $H$, then $f(\mu) \in LS_H( f(\nu) )$.

**Theorem 5.4.** Let $\sigma \in L^H$ and $\gamma \in LS(H)$ and $\sigma \subseteq \gamma$. If $\sigma \in LS_H(\gamma)$ and $f$ is a *homomorphism* of $G$ into $H$, then $f^{-1}(\sigma) \in LS_G(f^{-1}(\gamma))$.

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